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BGRAPH -- A PROGRAM FOR BIPLOT MULTIVARIATE GRAPHICS. VERSION 1--ETC(U)
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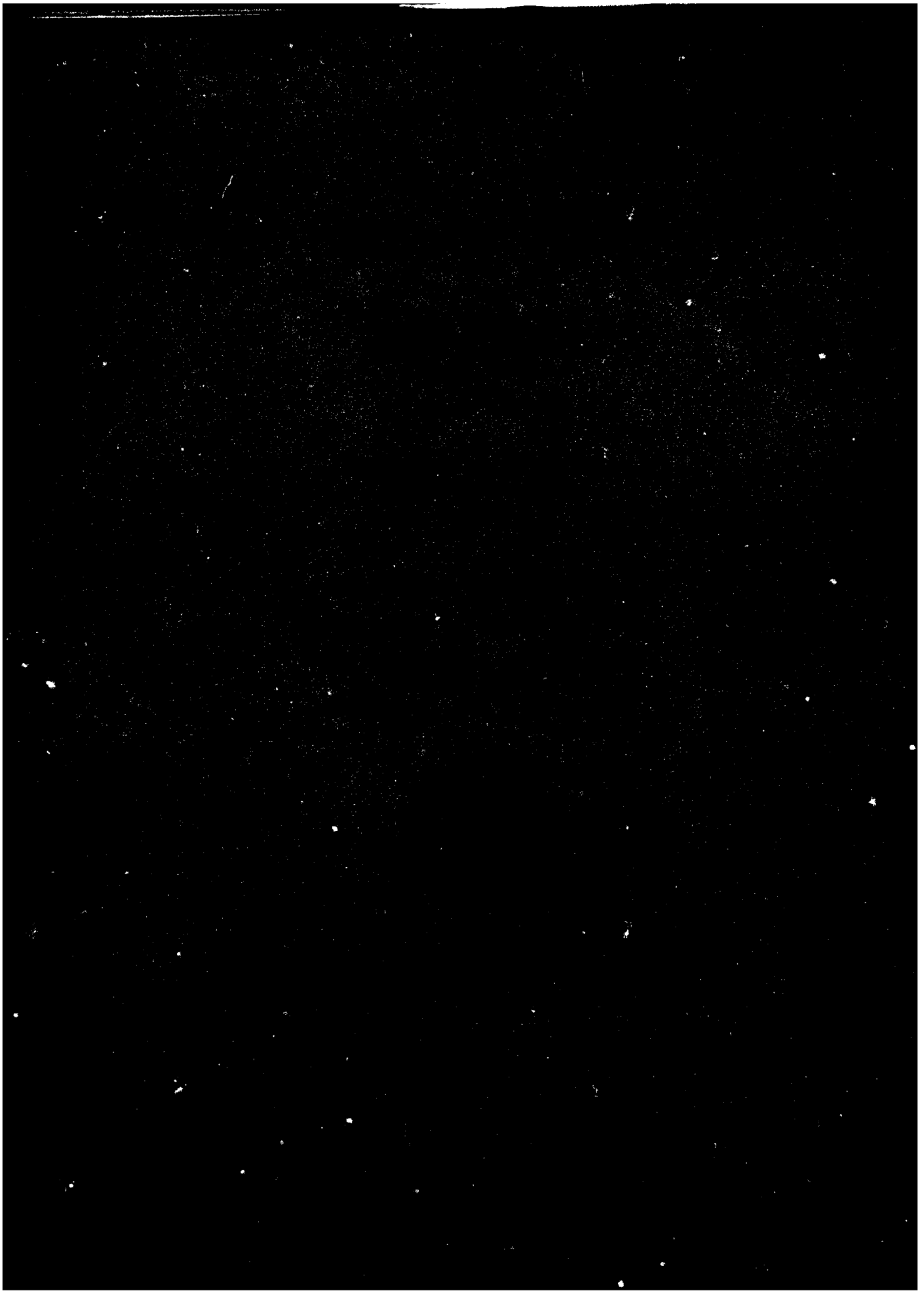

BGRAPH -- A Program for Biplot Multivariate Graphics.
Version 1: User's Guide

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1.0 Summary of BGRAPH Commands

Input and Output:	READ,ENTER, SAVE, NO SAVE,DISPLAY, NO DISPLAY.
Graphical Display of Two-Dimensional Biplots:	BIPLOT, MULTI
Graphical Display of Three-Dimensional Bimodels:	PERSPECTIVE, STEREO, SIDE, SIDE3, SIDE6
Rotating and Determining the Observer's Coordinate System:	ROTATE, FIRST, LAST OBSERVER, COORDINATES
Adjustment of Stereo Displays:	IOD, SHIFT, PLANE, VIEWPORT
Displaying Subsets of Row or Column Markers:	RPICK, ROMIT, RALL, RONLY, CPICK, COMIT, CALL, CONLY, BOTH
Labeling Graphical Displays:	TITLE, RLABEL, CLABEL, RPOINT, CPOINT, RVECTOR, CVECTOR, AXES, SCALE, FORMAT
Displaying Data Structure:	CIRCLE, ELLIPSE, MEANS
Changing Data Structure:	WINDOW, PROJECT, TRANSFORM, FACTOR
Special 3-D Viewing Commands:	COLOR, INTENSITY
System Commands:	HELP, DUMP, RESTORE, BRIEF, NO BRIEF, RESET, PRINT

2.0 Introduction

BGRAPH (Tsianco, 1980; Tsianco et. al., 1981) is an interactive, conversational program to perform biplot multivariate graphics. The program generates two- and three-dimensional biplot graphical displays based either on the singular value decomposition (SVD) of a matrix and the resulting rank 2 or 3 approximations or on any other input of biplot/bimodel coordinates. It displays various aspects of the biplot that have been found useful in visualizing data and in diagnosing models to fit data. The biplot and the bimodel have been described in a number of papers (Gabriel, 1971, 1972, 1978a, 1987b, 1980, 1981a, 1981b, 1981c, 1981d; Gabriel and Zamir, 1971; Gabriel et. al., 1974, 1976; Bradu and Gabriel, 1978; Corsten and Gabriel, 1976; Cox et. al., 1980; Cox and Gabriel, 1981; Strauss, Gabriel, et al., 1979). A very simple example is given next -- Section 2.1; a more detailed review of biplots, bimodels and their application is given in Section 3 below. A comprehensive example including the use of BGRAPH is presented in Section 7.

To biplot the n by m matrix Y, BGRAPH uses the singular value decomposition $Y = \Sigma_a \lambda_a \underline{p}_a \underline{q}_a'$ where $\lambda_a, \underline{p}_a, \underline{q}_a$ are, respectively, the a-th singular value, column and row. BGRAPH accepts input of r (up to six) components in the order $p_{1,1}$, $p_{2,1}, \cdots, p_{r,1}$; $p_{1,2}$, $p_{2,2}, \cdots, p_{r,2}; \cdots; p_{1,n}, p_{2,n}, \cdots, p_{r,n}$; $q_{1,1}$, $q_{2,1}, \cdots, q_{r,1}; q_{1,2}, q_{2,2}, \cdots q_{r,2}; \cdots; q_{1,m}, q_{2,m}, \cdots, q_{r,m}$; $\lambda_1, \lambda_2, \ldots, \lambda_r$; in (8G10.4) format. BGRAPH offers the use a

number of choices of two- or three-dimensional displays. Three-dimensional bimodels can be displayed as perspective plots, stereograms, analglyphs or as orthogonal views. A flexible set of commands offers the opportunity to manipulate the form of the graphical output, rotate displays, project, transform, label row and column markers, and to select subsets of row and column markers for display.

2.1 A Simple Example: A Biplot and its BGRAPH Display

A simple example of a biplot is given in Display 2.1. The 4 by 3 matrix Y can be factored as the product AB', A being 4 by 2, B' being 2 by 3. The biplot displays the rows of A, i.e., $\underline{a}_1'$, $\underline{a}_2'$, $\underline{a}_3'$ and $\underline{a}_4'$ as well as the rows of B, i.e., $\underline{b}_1'$, $\underline{b}_2'$ and $\underline{b}_3'$. The first row of A, i.e., the vector (2,2) is displayed as the point $\underline{a}_1$, the second row (2,1) is displayed as the point $\underline{a}_2$ and the other two rows as points $\underline{a}_3$ and $\underline{a}_4$. The columns of B' are displayed as arrows $\underline{b}_1$, $\underline{b}_2$ and $\underline{b}_3$. This distinction between arrow display for columns and point display for rows is convenient: the viewer immediately sees which are row markers and which are column markers.

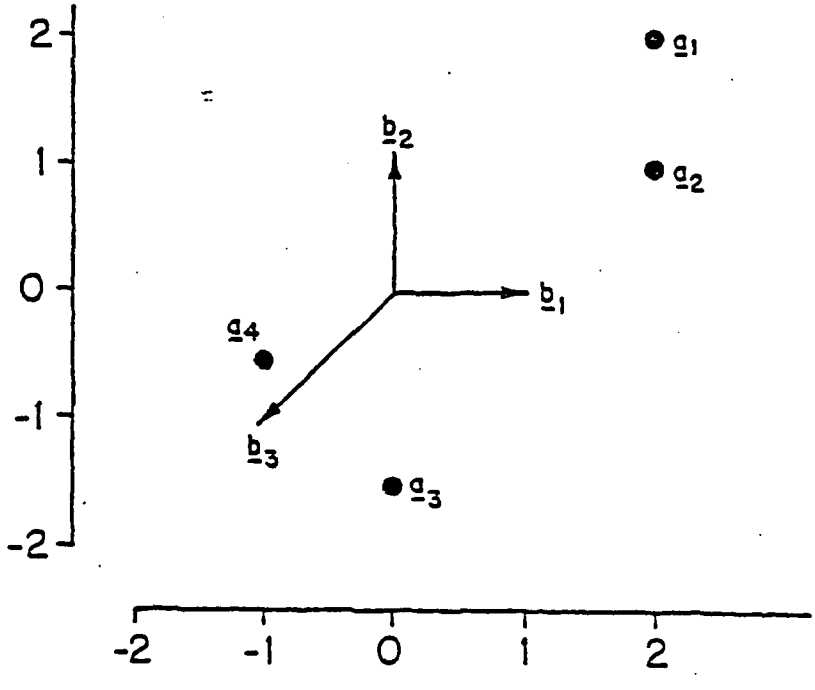
The inner product interpretation of this biplot can be seen from Display 2.2 which shows two of the elements of Y. Element $y_{2,3}$ is represented on the biplot by the inner product of $\underline{a}_2$ and $\underline{b}_3$. This inner product can be visualized by taking the direction through vector $\underline{b}_3$ and projecting the vector $\underline{a}_2$ onto it. The projection of $\underline{a}_2$ onto that direction

Display 2.1: A Biplot

Legend: { ● $\underline{a}_u$ is u-th row marker; ↗ $\underline{b}_v$ is v-th column marker }

$$Y = A B'$$

$$\begin{bmatrix} 2 & 2 & -4 \\ 2 & 1 & -3 \\ 0 & -1\tfrac{1}{2} & 1\tfrac{1}{2} \\ -1 & -\tfrac{1}{2} & 1\tfrac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 1 \\ 0 & -1\tfrac{1}{2} \\ -1 & -\tfrac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$



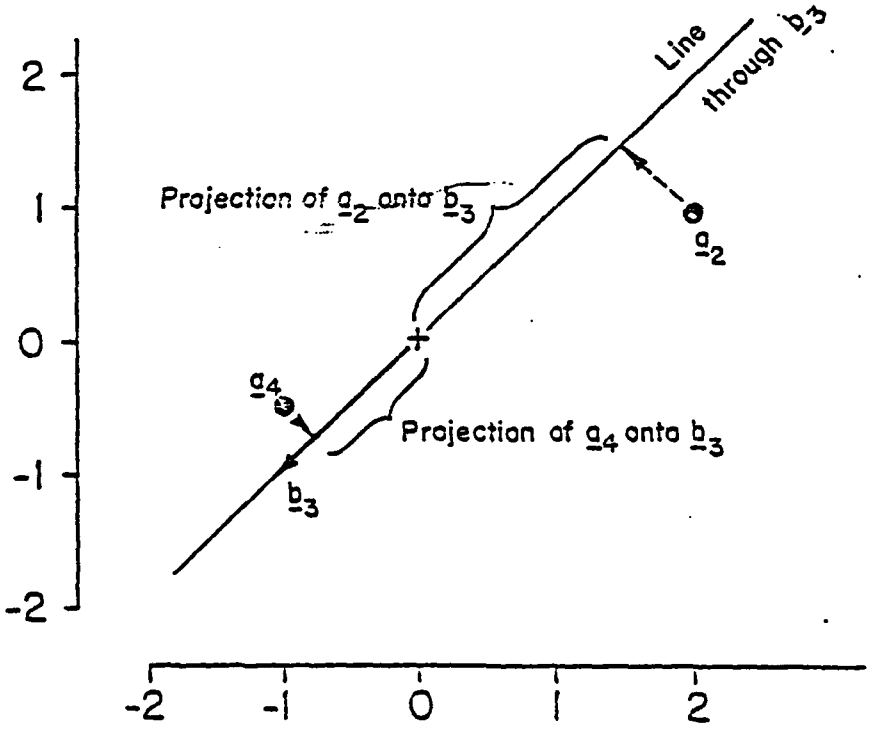
Display 2.2: Inner product representation of matrix elements on the biplot of Display 2.1.

$$\left\{ \begin{array}{l} y_{2,3} = -(\text{Length of } \underline{b}_3) \times (\text{Length of projection of } \underline{a}_2 \text{ onto } \underline{b}_3) \\ y_{4,3} = (\text{Length of } \underline{b}_3) \times (\text{Length of projection of } \underline{a}_4 \text{ onto } \underline{b}_3) \end{array} \right\}$$

Third column of Y

$$\underline{y}_{(3)} = A \, \underline{b}_3$$

$$\begin{bmatrix} -4 \\ -3 \\ 1\frac{1}{2} \\ 1\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 1 \\ 0 & -1\frac{1}{2} \\ -1 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

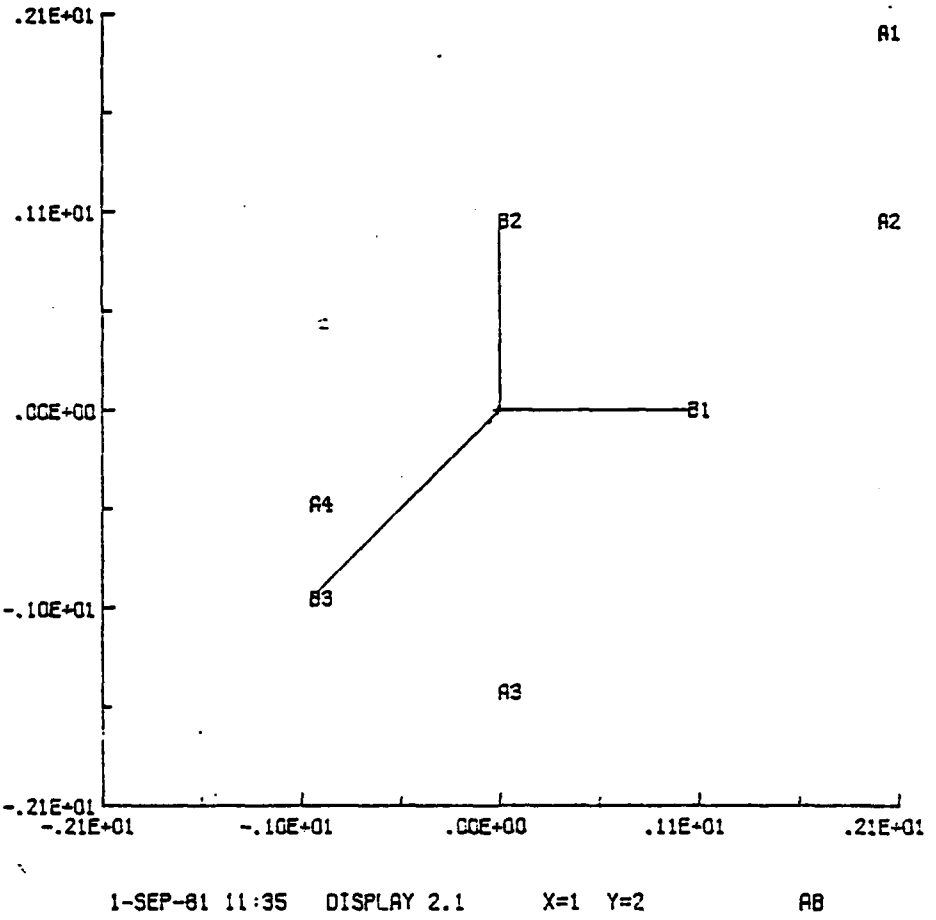


is $3/\sqrt{2}$ units long; the length of $\underline{b}_3$ itself is $\sqrt{2}$ units long; the product is $3/\sqrt{2} \times \sqrt{2} = 3$. Hence, the inner product is -3, the negative sign reflecting the projection's being in the direction opposite to that of the vector projected upon. Indeed, element $y_{2,3}$ is equal to -3. For another example, take element $y_{4,3}$. The inner product of $\underline{a}_4$ with b_3 is visualized by projecting $\underline{a}_4$ onto the direction through $\underline{b}_3$. (This is the same direction that was used before.) The projection is of length $3/2\sqrt{2}$; the vector projected onto is of length $\sqrt{2}$; they are both in the same direction; therefore, the inner product is $+3/2\sqrt{2} \times \sqrt{2} = 1\frac{1}{2}$ which is indeed the value of $y_{4,3}$.

This matrix Y could be biplotted exactly because it was of rank two. In general, an exact biplot of a matrix is possible only if the matrix is of rank one or two, because the biplot itself is planar. For a matrix of higher rank, several steps have to be taken in order to display it by an approximate biplot. The first step is to approximate the matrix Y by a matrix $Y_{[2]}$ of rank 2. The second step is to factor this rank 2 approximation $Y_{[2]}$ as a product AB' of a matrix $A_{(nx2)}$ and a matrix $B'_{(2xm)}$. The third step is to use each row of matrix A as a row marker $\underline{a}$ and each column of B' as a column marker $\underline{b}$. These markers are then plotted as an <u>approximate</u> biplot of the original matrix Y.

The display of this particular biplot by BGRAPH is shown in Display 2.3 which was generated by the commands ENTER, TITLE, CVECTOR, RLABEL, CLABEL, SCALE, BIPLOT and

Display 2.3: Display 2.1 generated on a Tektronix 4662 Plotter using BGRAPH.



Display 2.4: BGRAPH commands used to generate Display 2.3.

```
.RUN BGRAPH

ENTER TERMINAL TYPE CODE
0-DEC 340
1-COMPUTEK 400
2-ADAGE AGT-30
3-ARDS
4-TEKTRONIX 4010,4013
5-DEC GT40
6-TEKTRONIX 4014
7-OTHER

4                                   <----

B:ENTER                             <----

ENTER # OF POINTS IN 1ST SET: 4     <----
ENTER # OF POINTS IN 2ND SET: 3     <----
ENTER # OF COORDINATES PER POINT(MAX.= 6): 2       <----

TYPE 1 TO ENTER POINTS FROM TTY, OTHERWISE TYPE 0.

1                                   <----

ENTER F-FORMAT FOR READING POINTS.

(2F5.1)                             <----

ENTER COORDINATES IN 1ST SET, POINT BY POINT.

  2.0  2.0                         <----
  2.0  1.0                         <----
  0.0 -1.5                         <----
 -1.0 -0.5                         <----

ENTER COORDINATES IN 2ND SET, POINT BY POINT.

  1.0  0.0                         <----
  0.0  1.0                         <----
 -1.0 -1.0                         <----

B:TITLE                             <----

ENTER A TITLE UP TO 20 CHARACTERS LONG:  DISPLAY 2.1      <----

B:CVECTOR                           <----
COLUMN MARKERS WILL BE PLOTTED AS VECTORS.
```

Display 2.4 (continued)

```
B:RLABEL                              <----

CHOOSE LABELS FOR ROW MARKERS BY ENTERING:
1 FOR ROW NUMBER,
2 FOR RANKS ON SOME VARIABLE,
3 FOR DECILE RANKS MINUS 1(I.E. 0 TO 9),
4 FOR STANDARD SCORES ON SOME VARIABLE,
5 TO ENTER LABELS(MAX. LENGTH=5 CHARACTERS).

5                                     <----

ENTER 1 IF LABELS OR LABELING VARIABLE ARE TO BE
ENTERED FROM THE TTY, OTHERWISE ENTER 0.

1                                     <----

ENTER FORMAT FOR LABELS OR LABELING VARIABLE.
USE A-FORMAT)

(A2)                                  <----

ENTER LABELING VARIABLE OR LABELS(    4 VALUES OR
LABELS IN ALL):

A1                                    <----
A2                                    <----
A3                                    <----
A4                                    <----

B:CLABEL                              <----

CHOOSE LABELS FOR COLUMN MARKERS BY ENTERING:
1 FOR COLUMN NUMBER(PREFIXED BY C),
2 FOR RANKS ON SOME VARIABLE,
3 FOR DECILE RANKS MINUS 1(I.E. 0 TO 9),
4 FOR STANDARD SCORES ON SOME VARIABLE,
5 TO ENTER LABELS(MAX. LENGTH=5 CHARACTERS).

5                                     <----

ENTER 1 IF LABELS OR LABELING VARIABLE ARE TO BE
ENTERED FROM THE TTY, OTHERWISE ENTER 0.

1                                     <----

ENTER FORMAT FOR LABELS OR LABELING VARIABLE.
USE A-FORMAT)

(A2)                                  <----

ENTER LABELING VARIABLE OR LABELS(    3 VALUES OR
LABELS IN ALL):

B1                                    <----
B2                                    <----
B3                                    <----

B:SCALE                               <----

TYPE 0 TO ALLOW ROW AND COLUMN SCALES TO DIFFER;
OTHERWISE TYPE 1

1                                     <----


B:BIPLOT                              <----

        Note: This BIPLOT command produces Display 2.3

B:STOP                                <----
```

STOP. Display 2.4 gives the command sequence and responses to queries from BGRAPH. Input to BGRAPH is indicated by <----.

2.2 Description of BGRAPH -- How to Invoke BGRAPH on DEC 10.

BGRAPH displays two-and three-dimensional bimodels of data matrices $Y_{(nxm)}$ and can use their singular value decompositions (SVD). The program then operates on a suitably prepared disk data file referred to as a PLQ file. Creation of such a file is discussed in Section 5. When BGRAPH is invoked, i.e., by typing RUN BGRAPH, it will present a list of computer terminals for the user to select from. When the program is ready for terminal input, the user will be prompted with the characters B:. Any of the 58 BGRAPH commands described in Section 6.0 followed by a carriage return will then initiate action. A session can be terminated by issuing the STOP command in response to the prompt.

In the following, we briefly describe the types of commands available in BGRAPH. A dictionary of individual commands with detailed instructions appears in Section 6.

2.2.1 Input and Output

The command READ will cause BGRAPH to prompt for the name of a PLQ file and for a choice of one of the five standard biplot factorizations based directly on the SVD (See Section 3 and the command FACTOR in Section 6). An alternative form of input for other factorizations is provided by command ENTER.

Output is initiated by the graphical display commands. Graphs can be displayed on either the input terminal or sent to a disk data file for display on a secondary graphical output device. The commands SAVE, NO SAVE, DISPLAY and NO DISPLAY control the flow of graphical output.

2.2.2 Graphical Display of Two-Dimensional Biplots.

The BIPLOT command will plot the row and column markers. If unmodified by other commands, it displays the first two components of the markers. The MULTI command similarly displays biplots of all sets of pairs of components of the markers; it can do so for up to six components.

These two-dimensional BGRAPH plots are displayed in a square centered at the origin with sides scaled so that they are 2.1 times the largest (in absolute value) row marker component and 2.1 times the largest column marker component. The factor of 2.1 is chosen to allow all markers to be displayed and yet allow 5% extra margins for labels. The axes for row markers are at the bottom and left of the display; for the column markers, they are at the top and right.

2.2.3 Graphical Display of Three-Dimensional Bimodels

BGRAPH displays three-dimensional models by plotting orthogonal, oblique or perspective views of the row and column markers. Orthogonal projections onto selected planes

of observation are accomplished by the SIDE, SIDE3 and SIDE6 commands. The three displayed coordinates of the row and column markers are scaled to fit in a unit cube centered at the origin with each face orthogonal to one coordinate axis, i.e., going through the other two axes.

Oblique and perspective views represent three-dimensional objects on a plane by extending imaginary lines of sight from each object to an observation point (the location of the eye of the observer). Each view consists of the intersection of a plane and the lines of sight. In orthogonal views, the observation point is at infinity, so the lines of sight are essentially parallel but need not be perpendicular to the viewing plane. In perspective views, the observation point is no longer at infinity, so the lines of sight are no longer parallel.

The geometric and computational principles involved in producing 3D displays are outlined in Newman and Sproull (1980).

Perspective views of bimodels are generated by the PERSPECTIVE command; analglyphs and stereograms by the STEREO command.

Analglyphs and stereograms use pairs of perspective views to create three-dimensional images. Examples of analglyphs can be found in Pearce (1977), and examples of stereograms can be found in Fraser and Kovats (1966) and Rohlf (1971). One perspective view represents an object as the observer's left eye would see it and the other as the

observer's right eye would see it. The two approaches differ in the manner they display the two views. However, with each, the result is a pair of views which can be fused together to form a three-dimensional image.

In analglyphs, each view is displayed in a different color (typically red and green). The three-dimensional image is obtained when these are viewed through color filters -- so that the filter for the left eye filters out the view for the right eye and vice versa. The two views usually overlap considerably, but the color filters straighten out the apparent confusion.

In stereograms the two views are displayed side by side (i.e., with little or no overlap) and viewed through stereoscopic glasses. Color need play no part in a stereogram, although one might wish to use color for labeling. The optimal distance between the displays of the views depends on the characteristics of the glasses being used.

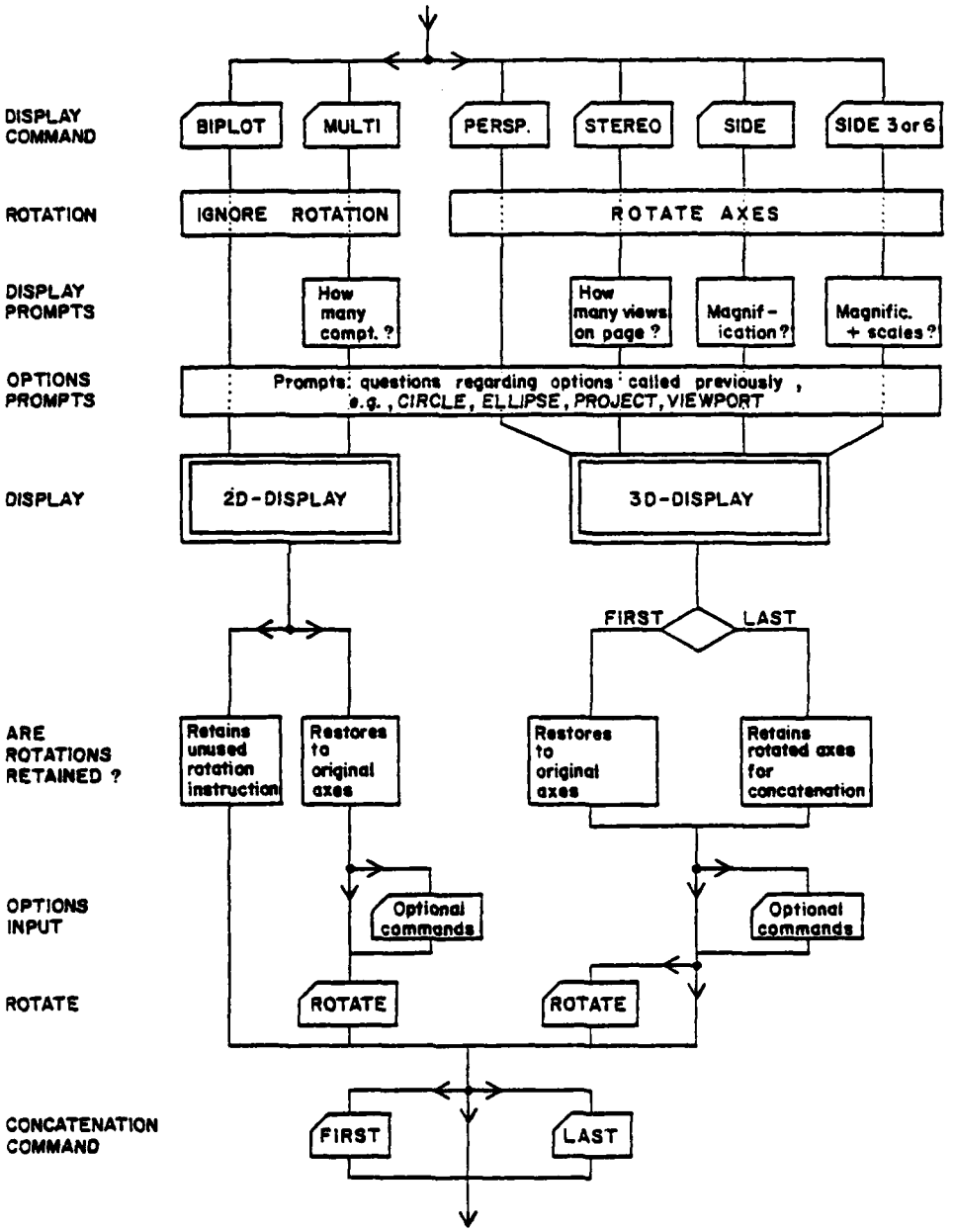
2.2.4 Rotation of Displays

Three-dimensional points representing row and column markers are scaled and translated to fit in a unit cube centered at the origin.

Any three of up to six components of the SVD can be assigned to the axes of the display in any order by the COORDINATES command. The unit cube of observation can be rotated about any axis through the center by using the ROTATE command. This command rotates the viewing cube

DISPLAY 2.5

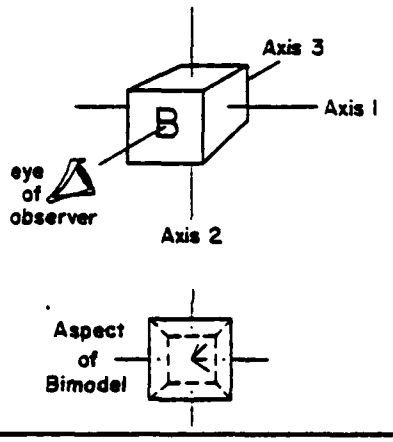
FLOWCHART OF DISPLAY AND ROTATE SEQUENCING



DISPLAY 2.6

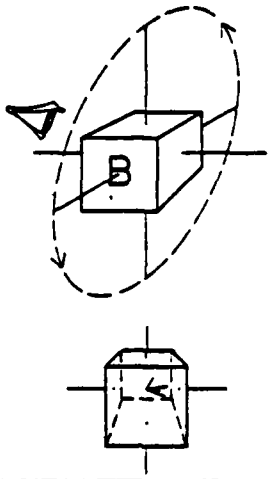
Effect of command ROTATE

INITIAL POSITION



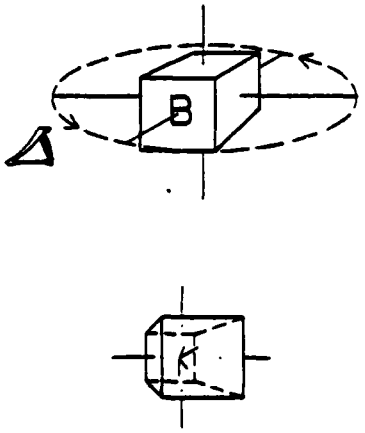
ROTATION ABOUT 1st AXIS

Commands 1 and 30 0 0
or 2 and 1 0 0 30



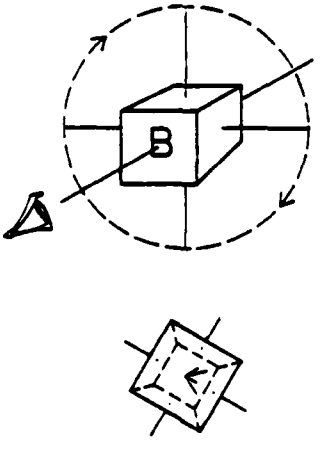
ROTATION ABOUT 2nd AXIS

Commands 1 and 0 30 0
or 2 and 0 1 0 30



ROTATION ABOUT 3rd AXIS

Commands 1 and 0 0 30
or 2 and 0 0 1 30



before display by the three-dimensional PERSPECTIVE, STEREO, SIDE, SIDE3 or SIDE6 commands. ROTATE has no effect on the two-dimensional display commands BIPLOT and MULTI -- See Display 2.5.

When the FIRST command is invoked (or at the beginning of a session or when RESET is invoked), the viewing cube is returned to standard orientation after each use of a three-dimensional display command. Invoking the LAST command before a three-dimensional display command will keep the unit cube in the position to which it was rotated and not return the cube to standard orientation. (However, a LAST command will be ineffective when followed by a BIPLOT or MULTI command before the next ROTATE -- and the cube will be returned to standard orientation.)

The orientation of the axes of the rotated cube with respect to the axes of standard orientation is reported in the lower left of each three-dimensional display. For guidance on the effect of various rotations see Figure 2.6.

2.2.5 Determining the Orientation and Scale of the Observer's Co-ordinate System with Respect to the Display

An alternative to rotating the cube containing the 3D bimodel is to change the point of observation with respect to the cube. OBSER sets that point by its three coordinates; its default is (0, 0, -3), i.e., facing the plane of the first two axes with a distance of 3 units from observation

point to origin. Some types of stereo display are enhanced by placing the observer slightly above the (0, 0, -3) location. The use of the OBSERVE command to rotate should be done with care. The authors have found it easy to lose themselves in three-space.

2.2.6 Adjusting Stereo Displays

BGRAPH allows selection of a variety of options which make it possible to produce three-dimensional displays with very little effort. For stereo pairs, it is possible to adjust both the imaginary inter-ocular distance of the observer (default value is 0.5), and the separation between displays of the left and right views (IOD, SHIFT). The user can choose to display stereo pairs on a single page or on separate pages, and can also decide how large a portion of the page a display is to occupy (VIEWPORT). The size of a perspective view is entirely dictated by the choice of viewport. (It would be more realistic if this were controlled by the distance between the cube, viewing plane and observation point.)

The PLANE, SHIFT and IOD commands can be used to tune stereograms to be viewed through different types of stereoscopic glasses. (The viewing plane should be between the cube and the observation point.) One can obtain maximum resolution by electing to draw stereo pairs on separate pages and using a copier to reduce them to a size appropriate for viewing. Stereographic glasses are available under the

name of Stereoscope from the Hubbard Scientific Company, Northbrook, Illinois. With glasses of 4.7 inches focal length and 2.2 times magnification, a viewport of 70% should be used on the Tektronix 4013; on the Tektronix 4662 a viewport of 50% is best. A good stereogram can also be produced by using the Tektronix 4662 with a viewport of 100% and then reducing the display using a 64% reduction followed by a 74% reduction with a Xerox 9200 copier. For stereograms the viewing plane should be between the observer and the cube. For analglyphs the viewing plane should be behind the cube.

Using the PLANE and SHIFT commands, it is possible to create analglyphs with the STEREO command. (The viewing of the plane should be behind the cube.) The input parameter d for the PLANE command can be used to place the plane of observation behind the cube and cause the observer's right eye to see the view normally seen by the left eye, and vice versa. The distance that the plane is placed from the origin depends on the position of the observer and on the cube's orientation. For the default position (0, 0, -3) and original orientation, this is obtained by choosing d = 1.167; for other positions and orientations, it needs to be chosen by trial and error. SHIFT will be set at 0 and IOD at 0.5 or a similar value to be fitted.

2.2.7 Displaying Subsets of Row and Column Markers

Subsets of row and column markers can be displayed on biplots through RPICK, ROMIT, RONLY, RALL, CPICK, COMIT, CONLY, CALL and BOTH commands. (See Dictionary -- Section 6.)

2.2.8 Labeling Graphical Output

The user can enter any labels for row and column markers by using the RLABEL and CLABEL commands. The default labeling is with the row number and with the column number. The RLABEL and CLABEL commands also provide the opporutnity for labels based on the rank of another variable, the decile ranks, or standard scores.

The RVECTOR and CVECTOR commands label rows or columns with a sheaf of lines from the origin. The AXES command suppresses printing of axes on two-dimensional biplots. The FORMAT command formats numerical labels on two-dimensional biplot axes. Markers can be displayed as numbered points by using RPOINT and CPOINT commands. If RLABEL and CLABEL are followed by input of blank "labels", only points will be displayed.

2.2.9 Display of Data Structure

The MEANS command displays the centroid of all row row markers and that of all column markers except those excluded by RPICK, ROMIT, CPICK or COMIT commands, respectively.

The ELLIPSE command asks for definition of groups of row markers (at most ten such groups). For each group it

computes two-dimensional or three-dimensional concentration ellipsoids (Cramer, 1946). These provide a convenient summary of the location and variability of the groups of markers.

On a biplot, or in a three-dimensional bimodel, every direction corresponds to a linear combination of the columns of the bimodelled matrix. Suppose that columns of the bimodelled matrix correspond to *different* variables. Then every direction on a biplot or three-dimensional bimodel corresponds to a linear combination of variables.

A concentration ellipse/ellipsoid contains points within k standard deviations of the centroid of the group's markers on every linear combination of variables. (BGRAPH prompts for choice of coefficient k.) The projection of the ellipse/ellipsoid onto a *given direction* shows the group mean on the corresponding linear combination of variables ±k times the group's standard deviation on that linear combination.

When the biplotted data consist of multivariate sample means -- one way MANOVA -- BGRAPH permits approximate testing of sample differences for significance. The command CIRCLE produces *comparison circles* (Gabriel, 1972) about each sample (row of Y) marker such that overlap (non-overlap) of the circles for two rows is to be interpreted as non-significance (significance) of the difference between the corresponding two samples.

The logic behind using such a display to test for a difference between samples is as follows. The Euclidean distance between row markers is an approximation to the numerator of the Hotelling T^2 test statistic one would use to make this comparison, while the sum of the radii of the circles centered at these two markers is an approximation to the denominator times the critical value for the test. (BGRAPH prompts for input of this value.) If these circles did not overlap, it would mean that the approximate critical value of the test statistic was greater than the approximate critical value for this test, and we would reject the hypothesis of no difference between samples.

In an m-dimensional bimodel, where m is less than the number of samples n, the approximation to the test statistic provided by the Euclidean distance between markers is an underestimate of the statistic. Therefore, if the number of samples is greater than two, the three-dimensional bimodel will usually provide a better approximate test than a biplot. Three-dimensional bimodels, use comparison spheres instead of circles. If the spheres fail to intersect, we conclude that the corresponding samples are significantly different.

The radius for a sample of size n_i is $\phi/\sqrt{2n_i}$ where ϕ^2 is a percentage point from the following distributions.

(a) For any individual T^2 test at level α, use $m/(n_e-m+1)$ times the upper α point of the F distribution with m and n_e-m+1 degrees of freedom.

(b) For Bonferroni-type simultaneous tests on all $k = \binom{m}{2}$ pairs of samples use the upper α/k point of the above distribution.

(c) For a maximum characteristic root STP at experiment-wise level α (Gabriel, 1968) use ϕ from the following tables and charts.

(1) For known variance-covariance matrix: From Choudhary-Hanumara and Thompson (1968) for $\tilde{p} = 2,\ldots,10$ or from Pillai and Chang (1969) for $\tilde{p} = 11,\ldots,20$ (The latter values should be multiplied by $\tilde{n}$.) where $\tilde{p} = \min(n - 1,m)$ and $\tilde{n} = \max(n - 1,m)$.

(2) For variance-covariance matrix estimated with n_e degrees of freedom, $\phi = n_e\,\theta/(1-\theta)$ where θ is read from Heck's (1960) charts or Pillai's (1960) tables with $\tilde{s} = \min(n - 1,m)$, $\tilde{m} = (|n-1-m|-1)/2$ and $\tilde{n} = (n_e-m-1)/2$ If sums of squares and products are input, rather than variance-covariance estimates, $n_e\theta/(1-\theta)$ must be replaced by $\theta/(1-\theta)$.

2.2.10 Changing Data Structure

BGRAPH's PROJECT command allows one to project extra points onto the row marker axes. One might do this to see if data on individuals not included in a biplotted data matrix differ from data on those included. In the AB' factorization, the axes for row marker display correspond to

the rows of B'. If a new data point is represented as an m-dimensional column vector $\underline{p}$, the coordinates of its projection onto these axes are the elements of $(B'B)^{-1} B'\underline{p}$. BGRAPH will read up to 100 new data points and project them onto the current row marker axes. Optionally, one can enter an m-dimensional vector of constants to be subtracted from the new data points before projection (e.g., if the bimodelled matrix consisted of deviations from column means and the new data points were raw scores, one would subtract column means). To save space, the new data points are not stored, but their projections are; as long as one continues to work with the same factorization, the projections remain available for use.

The WINDOW command moves and rescales the viewing window to enlarge or reduce a portion of the display. The command is useful for enlarging portions of a biplot where the markers are crowded.

The TRANSFORM command allows the user to specify an arbitrary linear transformation of the data before it is biplotted. The transformation carries the current bi-model factorization into a new factorization either by using the transformation directly or by a sequence of rotations, and changes of scale. The standard factorizations based on the SVD can be obtained by command FACTOR.

2.2.11 System Commands

The following system commands are available in BGRAPH to do bookkeeping functions:

HELP:	Displays list of commands and summary of options in effect.
DUMP:	Saves work session.
RESTORE:	Restores saved work session.
BRIEF:	Shortens output of system requests for user input.
NO BRIEF:	Cancels BRIEF command.
RESET:	Reset options to default values.
PRINT:	Stores on data file a log of graphical displays.
STOP:	Terminates work session and returns control to monitor.

2.2.12 Display of Plots on Secondary Output Devices

On the DEC 10 the SAVE command stores a metacode file that can be used (and reused) to display plots on hard-copy devices such as the Tektronix 4662 pen plotter. The programs T4662 and NCARV can be used to display output. See the DEC 10 system help file for details on T4662 and NCARV.

2.2.13 Magnifying or Reducing Displays

The WINDOW and VIEWPORT commands and a magnification option of the SIDE, SIDE3 and SIDE6 commands can be used to change the size of a biplot. VIEWPORT controls the size of

the entire display including the axes and the labels. It is useful primarily to adjust the size of stereo displays so that they will fit within the field of view of a pair of stereo glasses. (See Section 2.2.5 for advice on vieport size for output device.)

The WINDOW command chooses a center for the display anywhere in the cube (or square within the unit cube) with coordinates $x_i \leq \frac{1}{2}$, i=1,2,3. An edgelength for the cube $o < E < \infty$ can be specified. WINDOW magnifies the content of the cube 1/E times along each axes. The scale limits displayed on each axis are increased E-fold.

The magnification options within the SIDE, SIDE3 and SIDE6 commands have the same effect as WINDOW except that the display remains centered at the origin. For SIDE the magnification factor (<0) acts like 1/E, the same as the edgelength E in WINDOW. For SIDE3 and SIDE6 the magnification factor (<0) acts like E, the inverse of the edgelength E in WINDOW.

In the side commands 1) the points are scaled to fit in the viewing cube, 2) the cube and its contents are projected on a plane, 3) scales are chosen in the plane based upon the vertices of the cube. If the cube has been rotated, the points can occupy a small portion of the biplot. The magnification factor in the side commands can be used to correct for this distortion of a plot.

2.2.14 Interrelationships Between Commands

There are a number of interrelationships between commands. All commands except the eight system commands (HELP, DUMP, RESTORE, BRIEF, NO BRIEF, RESET, PRINT and STOP) directly influence the graphical output from the seven display commands (BIPLOT, MULTI, PERSPECTIVE, STEREO, SIDE, SIDE3 and SIDE6). Below are described several (not necessarily mutually exclusive) kinds of interrelationships between commands.

A. Commands that set options which are unchanged by other commands. These commands are not changed until they are invoked again or until a RESET command is issued.

READ, ENTER, COORDINATE, PLANE, TRANSFORM,
OBSERVER, IOD, SHIFT, TITLE, RLABEL, CLABEL,
AXES, FORMAT, MEANS, COLOR,

B. Commands which remain in effect until a counter-command is issued.

SAVE - NO SAVE, DISPLAY - NO DISPLAY,
FIRST - LAST, RONLY - BOTH, CONLY - BOTH,
ROMIT - RALL, RPICK - RALL, COMIT - CALL,
CPICK - CALL, BRIEF - NO BRIEF, DUMP - RESTORE.

C. Alternative commands no two of which should be used together.

ELLIPSE - CIRCLE

D. Commands which initiate input requests at the time the display commands (BIPLOT, MULTI, PERSPECTIVE, STEREO, SIDE, SIDE3 and SIDE6) are invoked.

WINDOW, ELLIPSE, TRANSFORM, PROJECT, COORDINATES,
VIEWPORT.

E. Commands which are reset by the RESET command.

AXES, BRIEF, CIRCLE, COLOR, COORDINATE, ELLIPSE,

LAST, MEANS, ROTATE, TRANSFORM, VIEWPORT, WINDOW.

F. Non-graphic display commands influenced by other commands.

ROTATE by FIRST or LAST.

TRANS by FIRST or LAST.

MEANS by RPICK, ROMIT, CPICK or COMIT.

3.0 Biplot Factorizations

BGRAPH can display any one of five factorizations of the SVD of matrix Y. The FACTORIZATION command (See Section 2.2.10) selects one of the factorizations in Display 3.1.

Display 3.1

Biplot Factorizations of $Y = P'\Lambda Q$

Name of Factorization	Factorization	Application
GH'	$P'(\Lambda Q)$	Principal Component Analysis
JK'	$(P'\Lambda)Q$	Contingency Table Analysis, MANOVA
GF'	$P'(\Lambda^{-1}Q)$	Moore-Penrose Inverse
EK'	$(P'\Lambda^{-1})Q$	
SQRT	$(P'\Lambda^{\frac{1}{2}})(\Lambda^{\frac{1}{2}}Q)$	Variance-Covariance Matrices

All these factorizations provide lower rank approximations to data matrices Y. The best known method for lower rank approximation is due to Householder and Young (1938). For example, for rank 2 it minimizes the sum of squares of the deviations of elements of Y from elements of the reduced rank matrix $Y_{[2]}$. This method cannot be applied directly when weights are involved. The elegant mathematical relations that were used by Householder and Young break down as soon as one uses weighted least squares and multiplies the

squared deviation $(y_{i,j}-y_{[2]i,j})^2$ by a weight $w_{i,j}$. However, an algorithm is available (Gabriel and Zamir, 1979), which allows this more general approximation. (For a special kind of weights, Haber (1975) found an earlier solution.) Another method of fitting lower rank matrices is by adaptive fits (McNeil and Tukey, 1975) and yet further methods are being studied.

Factorization of the rank 2 approximation $Y_{[2]}$ is always possible. Matrices $A_{(nx2)}$ and $B_{(mx2)}$ that satisfy $Y_{[2]} = AB'$ must exist. That follows from the definition of the rank of a matrix. However, such a factorization is not unique. In fact, if we post-multiply A by any 2 x 2 non-singular R and pre-multiply B' by the inverse R^{-1}, the resulting $(AR)(R^{-1}B)$ factorization is just as valid as the original AB' factorization. We, therefore, have a choice as to which factorization to biplot. Note that transformation by a non-singular matrix consists of a rotation of axes, a scaling along the new axes and another rotation, whereas the transformation by the inverse consists of the same rotations with a scaling which is reciprocal to the first one. This may help to give an idea of how different factorizations and different biplots are related. (An illustration of alternative factorizations and the resulting biplots was given by Gabriel, 1971.)

The non-uniqueness of the factorization has some advantages for the statistician, who may choose a factorization which has desirable data analytic or statistical features.

For instance, one particulary attractive factorization is referred to as the GH' factorization. This has orthonormal columns for G and therefore satisfies Y'Y = HH', and is especially useful if the rows of Y represent individuals and the columns represent variables. Then Y'Y is n times the estimated variance-covariance matrix, and so the inner products of the rows $\underline{h}$ of H in a GH' biplot represent the covariances, and the squared lenghts of the $\underline{h}$'s represent the variances. Furthermore, the cosines of angles between $\underline{h}$-vectors represent the correlations between the variables. This GH' biplot is useful in many statistical applications.

The JK' factorization is a useful alternative in which the statistical distances between rows of Y are closely approximated by those between j markers. The resulting JK' biplot has often been used in presenting principal component analyses where comparison of individuals (row of Y) was of interest, rather than a study of variables (columns of Y) (Greenacre, 1980).

In particular, when the elements of Y are the m-variate means of m samples (the one-way MANOVA situation), the JK' factorization allows multivariate comparison of samples by means of the j-markers. The j-distances approximate Mahalanobis distances and may be used for graphical approximations to Hotelling T^2 tests.

A special case occurs when Y comes from a contingency table by dividing each row by the row total. The resulting row of proportions can be regarded as multivariate means and

MANOVA methods can be applied. The j-markers permit chi-square-type comparisons of samples (rows of Y).

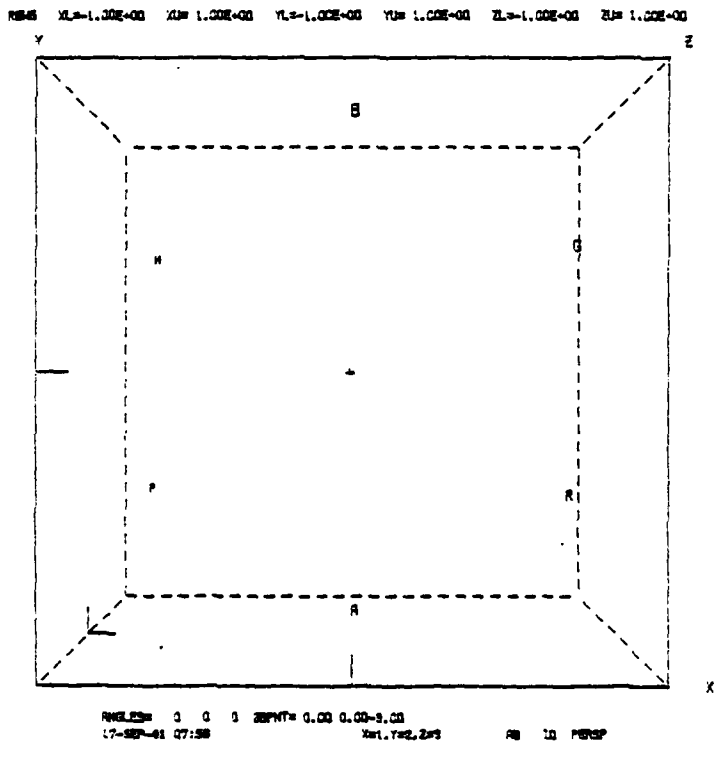
Further applications are indicated in Display 3.1.

4.0 Implementation of BGRAPH

BGRAPH (Tsianco, 1980) is written in FORTRAN and is implemented on a DEC 10 under the TOPS 10 operating system. Graphic display is carried out through the system plot package of the National Center for Atmospheric Research (NCAR) graphics package. Graphics display can be on any device which is supported by the NCAR package.

The NCAR subroutines utilized by BGRAPH perform only low level functions such as point and line drawing so such functions from other graphics packages could be substituted.

The singular value decomposition of the data matrix required by BGRAPH is not performed within the program. See Section 5 and Section 7 for details on preparing input for BGRAPH.

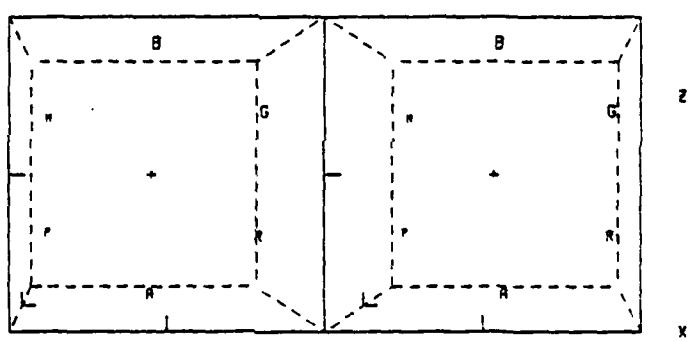

XL=-1.00E+00 XU= 1.00E+00 YL=-1.00E+00 YU= 1.00E+00 ZL=-1.00E+00 ZU= 1.00E+00
Y
Z
B
H
G
P
R
A
X

5.0 Preparing Data for Input to BGRAPH: BIPLOT, SVD

BGRAPH performs the graphical display of the decomposition of a data matrix. There are many suitable decompositions, so BGRAPH is designed to accept singular value decompositions via the READ command and other decompositions via the ENTER command.

At the University of Rochester two programs are available to perform the appropriate decomposition: BIPLOT [5370,5370] and SVD [5370,5370]. Examples of the use of BIPLOT and SVD are given in Section 7.2.

RB46 XL=-1.00E+00 XU= 1.00E+00 YL=-1.00E+00 YU= 1.00E+00 ZL=-1.00E+00 ZU= 1.00E+00
Y Z Y



ANGLES= 0 0 0 BBPNT= 0.00 0.00-5.00
17-SEP-81 07:58 X=1,Y=2,Z=3 AB :D STEREO

6.0 Dictionary of BGRAPH Commands.

In this section, each command is described and an example is given of the response to the BGRAPH prompt B:. Input is indicated by an arrow <----. BGRAPH output which is displayed on the input terminal is also shown.

Some commands are executed only when the next display commmand is invoked. The operation invoked by such commands is therefore described with the verb "to be" rather than "is" or "are". (See, for example, CALL.)

AXES

Supresses printing of axes on two-dimensional biplot displays.

```
B:AXES                                    <----
```

Reference: Sections 2.2.8, 2.2.14; Display 7.7.

BIPLOT

Displays two-dimensional bimodels.

```
B:BIPLOT                                  <----
```

Reference: Sections 2.2.2, 2.2.4; Displays 2.4, 7.4, 7.5, 7.7, 7.12.

BOTH

Cancels RONLY or CONLY command and causes both row and column markers to appear on two-or three-dimensional bimodel displays.

```
B:BOTH                                    <----

BOTH ROW AND COLUMN MARKERS WILL BE OPERATED ON.
```

Reference: Sections 2.2.7, 2.2.14.

BRIEF

Shortens output of system requests for user input. See also NO BRIEF.

```
B:BRIEF                                    <----
```

Reference: Sections 2.2.11, 2.2.14.

CALL

Cancels COMIT and/or CPICK and causes all column markers to be displayed.

```
B:CALL                                     <----

ALL COLUMN MARKERS WILL BE INCLUDED WHEN
COLUMN MARKERS ARE EMPLOYED.
```

Reference: Sections 2.2.7, 2.2.14.

CIRCLE

Comparison *circles or spheres* to appear when MANOVA biplot display is called. (Do not use when ELLIPSE is invoked.) After a display command, program prompts for radii of circle.

```
B:CIRCLE                                   <----

TYPE 1 TO DRAW CIRCLES OR SPHERES, OR 0 TO OMIT
THEM.

1                                          <----
```

Reference: Sections 2.2.9, 2.2.14; Display 7.12.

CLABEL

Labels column markers for future displays. Default has labels C1, C2, ... RESET returns to this default.

```
B:CLABEL                                   <----
```

Reference: Sections 2.2.8, 2.2.14; Displays 7.4, 7.12.

COLOR

Specifies colors for stereo pairs. (Not implemented on the DEC 10.)

B:COLOR <----

TYPE 1 TO SELECT COLOR(S), OR 0 TO USE THE DEFAULT.

1 <----

TYPE 2 TO SELECT COLORS FOR THE TWO VIEWS IN AN ANALGLYPH, OTHERWISE, TYPE 1.

1 <----

ENTER 1 3 CHARACTER COLOR CODE(S) IN 1A3 FORMAT.

007

Reference: Section 2.2.14.

COMIT

Omits a subset of column markers from future displays. (Cancelled by CALL.)

B:COMIT <----

ENTER LIST OF COLUMN NUMBERS TO BE EXCLUDED (3 - 5 = 3,4,5). END WITH A /.

Reference: Sections 2.2.7, 2.2.9, 2.2.14.

CONLY

Column markers are displayed but not row markers. (Cancelled by BOTH command.)

B:CONLY <----

ONLY COLUMN MARKERS WILL BE OPERATED ON.

Reference: Sections 2.2.7, 2.2.14.

COORDINATES

Assigns components of the SVD to axes of the graphical displays. This assignment remains in effect until the next COORDINATE command or RESET. Default assigns components 1, 2, 3 to axes X, Y, Z, respectively.

B:COORDINATES <----

THE NEXT PLOT COMMAND WILL REQUEST COORDINATES.

Reference: Sections 2.2.4, 2.2.14.

CPICK

Picks a subset of column markers for future displays. (Cancelled by CALL command.)

```
B:CPICK                                    <----

ENTER LIST OF COLUMN NUMBERS TO BE INCLUDED
(3-5=3,4,5).  END WITH A /.

1 3/                                       <----
```

Reference: Sections 2.2.7, 2.2.9, 2.2.4.

CPOINT

Cancels CVECTOR and causes column markers to be displayed as points.

```
B:CPOINT                                   <----

COLUMN MARKERS WILL BE PLOTTED AS POINTS.
```

Reference: Section 2.2.8.

CVECTOR

Column markers to be displayed as vectors emanating from the origin. Reset to CPOINT by RESET. (If WINDOW excludes origin, or endpoint of a vector, these vectors are not drawn.)

```
B:CVECTOR                                  <----

COLUMN MARKERS WILL BE PLOTTED AS VECTORS.
```

Reference: Section 2.2.8; Displays 7.4, 7.12.

DISPLAY

Graphical output from two-dimensional and three dimensional graphical commands will be displayed on input terminal. Reset by NO DISPLAY command. Input terminal must be able to display graphical output: Tektronix 4013, 4014, 4015, DEC 340, GT40, COMPUTEK 400, ADAGE AGT-30, ARDS.

```
B:DISPLAY                                    <----

ALL SUCCEEDING PLOTS WILL BE DISPLAYED UNTIL THE NO
DISPLAY COMMAND IS GIVEN.
```

Reference: Sections 2.2.1, 2.2.14.

DUMP

Saves work session on disk data file. DUMP does not change any system options. Use RESTORE to restore saved work session.

```
B:DUMP                                       <----

Unit=44:  /ACCESS=SEQINOU/MODE=IMAGE

Enter new file specs.  End with an $(ALT)

     *EXAMPLE.DMP$                           <----

DUMP TO FILE COMPLETED.
```

Reference: Sections 2.2.11, 2.2.14.

ELLIPSE:

Concentration ellipsoids calculated for subsets of row markers. (The subset definitions are prompted after a display command is given, as are subset labels and number of standard deviations or standard errors encompassed by ellipses.)

```
B:ELLIPSE                                    <----

TYPE 1 TO DRAW CONCENTRATION ELLIPSOIDS, OR 0 TO
OMIT THEM.

1                                            <----

TYPE 1 TO PLOT ROW MARKERS AS WELL,
     2 TO EXCLUDE MARKERS INSIDE THEIR RESPECTIVE
       ELLIPSOIDS, OR
     3 TO SUPPRESS PLOTTING OF ALL ROW MARKERS.

3                                            <----
```

Reference: Sections 2.2.9, 2.2.14; Display 7.7.

ENTER

Requests data to be input directly from the terminal or from a disk data file. Allows any decomposition Y=AB' to be input for graphical display. All defaults are reset by ENTER command.

```
B:ENTER                                    <----
```

Reference: Sections 2.2.1, 2.2.14, 5.0; Display 2.4.

FACTORIZATION

Sets factorization for bimodel based on SVD. (ENTER command would specify factorization not based on SVD.)

```
B:FACTORIZATION                            <----

ENTER 1 FOR GH FACTORIZATION,
      2 FOR JK FACTORIZATION,
      3 FOR GF FACTORIZATION,
      4 FOR EK FACTORIZATION,
      5 FOR SQRT FACTORIZATION.

1                                          <----
```

Reference: Section 2.2.10.

FIRST

Causes ROTATE command to rotate from standard orientation and/or TRANS command to transform from identity. Changed by LAST command.

```
B:FIRST                                    <----

TRANSFORMATIONS WILL NOT BE CONCATENATED.
```

Reference: Sections 2.2.4, 2.2.5, 2.2.14; Display 2.5.

FORMAT

Formats numerical labels on biplot axes.

```
B:FORMAT                                   <----

FORMAT SPECIFICATION MUST BE LE 10 CHARACTERS LONG,
INCLUDING PARENTHESES, AND BE IN F OR E FORMAT.
ENTER FORMAT FOR ROW MARKER HORIZONTAL AXIS LABELS.
```

```
(F2.0)                                    <----

ENTER FORMAT FOR ROW MARKER VERTICAL AXIS LABELS,
OR R TO REPEAT THE PRECEEDING FORMAT.

R                                         <----

ENTER FORMAT FOR COLUMN MARKER HORIZONTAL AXIS
LABELS, OR R TO REPEAT THE PRECEEDING FORMAT.

R                                         <----

ENTER FORMAT FOR COLUMN MARKER VERTICAL AXIS LABELS,
OR R TO REPEAT THE PRECEEDING FORMAT.

R                                         <----
```

Reference: Sections 2.2.8, 2.2.14.

HELP

Displays list of BGRAPH commands and a summary of options in effect.

```
B:HELP                                    <----

PROGRAM STATUS REPORT:

TITLE:  DISPLAY 3.1

OPTION SETTINGS:

  AXES  BRIEF  CIRCL  COLOR  COORD  ELLIP  INTEN  LAST  MEANS
    ON    OFF    OFF     ON    OFF    OFF     ON   OFF    OFF

 PRINT  PROJE  ROTAT  SAVE  SCALE  TRANS  VIEWP  WINDO
   OFF    OFF    OFF   OFF    OFF    OFF     ON    OFF

                SUBSET  PLOT  POINT

          ROWS:    OFF    ON    OFF
          COLS:    OFF    ON    OFF

          FACTORIZATION: AB  X=1, Y=2, Z=3  OBPNT= 0.00 0.00-3.00

          WINDOW CENTER=  0.00 0.00 0.00  EDGELENGTH=  1.00

          3D VIEWPORT %=  100.
```

COMMANDS AVAILABLE IN BGRAPH:

PRINT	NO DI	TITLE	PLANE	CALL	BIPLO	RONLY	CIRCL	COMIT
OBSER	CPICK	IOD	RLABE	FACTO	STERE	WINDO	SAVE	COORD
DUMP	FIRST	FORMA	RVECT	MULTI	CONLY	PERSP	BOTH	ROMIT
NO BR	CLABE	HELP	RESET	RESTO	MEANS	DISPL	RPICK	ELLIP
SHIFT	COLOR	VIEWP	PROJE	CPOIN	STOP	ENTER	BRIEF	TRANS
AXES	SIDE3	SCALE	RALL	RPOIN	INTEN	READ	SIDE6	NO SA
SIDE	ROTAT	LAST	CVECT					

Reference: Sections 2.2.11, 2.2.14.

INTENSITY

Intensity of points is made inversely proportional to squared distance from observation point.
(Not implemented on DEC 10.)

B:INTENSITY <----

TYPE 1 TO VARY INTENSITY IN 3D-PLOTS, OTHERWISE TYPE 0.

1 <----

Reference:

IOD

Adjust inter-ocular distance for viewing stereographs.

B:IOD <----

TYPE 1 TO ENTER A NEW INTEROCULAR DISTANCE, OR
0 TO USE THE DEFAULT.

1 <----

ENTER NEW INTEROCULAR DISTANCE. SEE MANUAL FOR DETAILS.

.5 <----

Reference: Sections 2.2.6, 2.2.14.

LAST

Causes ROTATE and/or TRANS to function from the last position specified before a 3-D display.

```
B:LAST                                    <----

TRANSFORMATIONS WILL BE CONCATENATED.
```

Reference: Sections 2.2.5, 2.2.14.

MEANS

Calculates centroids of the row markers and the column markers which were not omitted by RPICK, ROMIT, CPICK and/or COMIT.

```
B:MEANS                                   <----

TYPE 1 TO PLOT MARKER MEANS, OR
     0 TO OMIT THEM.

1                                         <----
```

Reference: Sections 2.2.9., 2.2.14; Display 7.10.

MULTI

Displays of two-dimensional bimodel for all pairs of the first components, (k chosen upon prompting).

```
B:MULTI                                   <----

ENTER NUMBER OF COMPONENTS TO BE USED

3                                         <----
```

Reference: Section 2.2.2, 2.2.4; Display 7.11.

NO BRIEF

Cancels BRIEF command.

```
B:NO BRIEF                                <----
```

Reference: Sections 2.2.11, 2.2.14.

NO DISPLAY

Cancel the DISPLAY command -- Graphical output will not be displayed on the input terminal.

```
B:NO DISPLAY                              <----

NO PLOTS WILL BE DISPLAYED UNTIL THE DISPLAY
COMMAND IS GIVEN.
```

Reference: Sections 2.2.1, 2.2.14.

NO SAVE

Cancels SAVE command -- Graphical output will not be saved as a metacode file for output to secondary display device.

```
B:NO SAVE                                    <----

NO PLOTS WILL BE SAVED UNTIL THE SAVE COMMAND IS GIVEN.
```

Reference: Sections 2.2.1, 2.2.14.

OBSERVER

Places the position of the observer. The default position is (0, 0, -3). Any point more than $\sqrt{3}$ cube units from origin may be chosen. The viewing plane is perpendicular to the line connecting the observer and the origin.

```
B:OBSERVER                                   <----

TYPE 1 TO ENTER AN OBSERVATION POINT, OR
     0 TO USE THE DEFAULT.

1                                            <----

ENTER COORDINATES OF THE NEW OBSERVATION POINT
(MUST BE AT LEAST SQRT(3) CUBE UNITS FROM THE ORIGIN.)

0 0 3                                        <----
```

Reference: Sections 2.2.5, 2.2.14; Displays 7.6, 7.8, 7.9.

PERSPECTIVE

Displays a perspective projection of three-dimensional bimodel onto the observer's viewing plane (as defined by observer).

```
B:PERSPECTIVE                                <----
```

Reference: Sections 2.2.3, 2.2.4; Displays 7.6, 7.8, 7.12.

PLANE

Adjustment of stereo displays. Adjusts the distance of the observer's viewing plane from the observation point. For analglyphs, set plane to 1.167 if observer position and cube orientation are at their defaults.

```
B:PLANE

TYPE 1 TO SELECT THE VIEWING PLANE, OR
     0 TO USE THE DEFAULT.

1                                        <----

ENTER A NUMBER GREATER THAN 0. SEE MANUAL FOR
EXPLANATION.

.5                                       <----
```

Reference: Sections 2.2.6, 2.2.14.

PRINT

Stores a log of graphical displays from current work session on a disk data file (not implemented).

```
B:PRINT                                  <----

ENTER PRINT CODE

     0 - NOTHING PRINTED.
     1 - PRINT DATE, TIME AND OPTIONS SELECTED
     2 - PRINT ABOVE PLUS TRANSFORMATION SUMMARY
     3 - PRINT ABOVE PLUS PROJECTION SUMMARY
     4 - PRINT ABOVE PLUS CONCENTRATION ELLIPSOID
         SUMMARY
     5 - PRINT ABOVE PLUS MARKER COORDINATES

5                                        <----

Unit=43   :/ACCESS=SEQINOU/MODE=ASCII

Enter new file specs.  End with an $(ALT).

*PRINT.LOG$
```

Reference: Sections 2.2.11, 2.2.14.

PROJECT

Allows projections of up to 100 new observations on the axes of the row markers. After the display command, it prompts for format of observations and for translation to another origin.

```
B:PROJECT                                    <----

TYPE 1 TO PROJECT NEW DATA POINTS, OTHERWISE TYPE 0.

1                                            <----
```

Reference: Sections 2.2.10, 2.2.14; Display 7.12.

RALL

Cancels the effect of ROMIT and/or RPICK and displays all row markers.

```
B:RALL                                       <----
```

Reference: Sections 2.2.7, 2.2.14; Display 7.11.

RLABEL

Labels row markers for future displays. Default to labels 1, 2, ... Reset returns to these labels.

```
B:RLABEL                                     <----
```

Reference: Sections 2.2.8, 2.2.14; Display 7.4.

READ

Requests name of PLQ data file containing singular value decomposition of data matrix. Requests biplot factorization. All defaults are reset by READ command. The READ command reads r components of the SVD. Use programs BIPLOT or SVD to perform singular r value decomposition ($r \leq 6$). Biplot factorization can also be controlled by FACTORIZATION command.

```
B:READ                                       <----
```

Reference: Sections 2.2.1, 2.2.14, 5.0; Displays 7.4, 7.12.

RESET

Resets system options to default values for the following commands: AXES, BRIEF, CIRCLE, COLOR, COORD, ELLIPSE, LAST, MEANS, PRINT, ROTATE, TRANSFORM, VIEWPORT, WINDOW. See HELP command.

```
B:RESET                                    <----

RESET COMPLETED.
```

Reference: Sections 2.2.4, 2.2.11, 2.2.14.

RESTORE

Restores work session saved by DUMP command.

```
B:RESTORE                                  <----

Unit=44  :/ACCESS=SEQINOU/MODE=IMAGE

Enter new file specs.  End with an $(ALT).

*EXAMPLE.DMP$

RESTORED FILE WAS DUMPED ON  3-AUG-81 AT 10:58
PROGRAM STATUS REPORT:
TITLE:  DISPLAY 3.1
OPTION SETTINGS:

AXES  BRIEF  CIRCL  COLOR  COORD  ELLIP  INTEN  LAST  MEANS
  ON    OFF    OFF     ON    OFF    OFF     ON   OFF    OFF
PRINT PROJE  ROTAT  SAVE   SCALE  TRANS  VIEWP  WINDO
  OFF   OFF    OFF    OFF    OFF    OFF     ON   OFF
        SUBSET  PLOT  POINT
ROWS:   OFF     ON    OFF
COLS:   OFF     ON    OFF
FACTORIZATION:  AB     X=1, Y=2, Z=3  OBPNT= 0.00 0.00 -3.00
WINDOW CENTER=  0.00 0.00 0.00   EDGELENGTH= 1.00
3D VIEWPORT %=  100.
```

Reference: Sections 2.2.11, 2.2.14.

ROMIT

Omits a subset of row markers from future displays. (Cancelled by RALL.)

```
B:ROMIT                                    <----

ALL ROW MARKERS WILL BE INCLUDED WHEN ROW MARKERS
ARE EMPLOYED.
```

Reference: Sections 2.2.7, 2.2.9, 2.2.14.

RONLY

Row markers are displayed but not column markers. (Cancelled by BOTH command.)

```
B:RONLY                                    <----

ONLY ROW MARKERS WILL BE OPERATED ON.
```

Reference: Sections 2.2.7, 2.2.14.

ROTATE

Rotates bimodel before 3D display. the rotation will be concatenated to previously 3D-displayed roations if LAST is on. They will not be concatenated if FIRST is on or if previous ROTATE was not used in a 3D display. The program prompts for choice of three angles or one axis and one angle for rotation.

```
B:ROTATE                                   <----

TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR
     2 TO GIVE AN AXIS AND AN ANGLE.

1                                          <----

ENTER 3 ANGLES (IN DEGREES) TO ROTATE THE CUBE
COUNTER-CLOCKWISE ABOUT AXES STHROUGH THE CENTER OF
THE CUBE, FIRST ABOUT THE X-AXIS, THEN THE Y-AXIS
AND FINALLY THE Z-AXIS.

0 0 60                                     <----

B:ROTATE

TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR
     2 TO GIVE AN AXIS AND AN ANGLE.

2                                          <----

TO ROTATE THE CUBE COUNTER-CLOCKWISE ABOUT AN AXIS
THROUGH THROUGH THE CENTER OF THE CUBE, ENTER A
VECSTOR TO DEFINE THE AXIS AND AN ANGLE (IN DEGREES).

1 0 0                                      <----
 60                                        <----
```

Reference: Sections 2.2.4, 2.2.5, 2.2.14; Displays 2.5, 6.1, 7.6, 7.8, 7.9, 7.12, 7.14, 7.15.

NOTE: Counterclockwise means seen from position on negative axis, i.e., for X on left, for Y below, for Z in front. These are the axes of the display, i.e., horizontal, vertical and depth; they are not the axes of the box shown around the bimodel. (Error: Following commands 2 1 0 0, the angle may need a negative sign -- see Display 2.6.)

RPICK

Picks a subset of row markers, for future displays. (Cancelled by the RALL commmand.)

Format:

B:RPICK <----

ENTER LIST OF ROW NUMBERS TO BE INCLUDED (3-5=3, 4, 5).

END WITH A /.

3 4/ <----

Reference: Sections 2.2.7, 2.2.9, 2.2.14; Display 7.10.

RPOINT

Cancels RVECTOR and causes row markers to be displayed as unlabelled points.

B:RPOINT <----

ROW MARKERS WILL BE PLOTTED AS POINTS.

Reference: Section 2.2.8.

RVECTOR

Row markers to be displayed as vectors emanating from the origin.

B:RVECTOR <----

ROW MARKERS WILL BE PLOTTED AS VECTORS.

Reference: Section 2.2.8.

SAVE

Stores graphical output on data file for display on a hard copy device. Graphical output from BIPLOT, MULTI, PERSPECTIVE, STEREO, SIDE, SIDE3 and SIDE6 are stored on disk data file.

```
B:SAVE                                          <----

ALL SUCCEEDING PLOTS WILL BE SAVED UNTIL NO SAVE
COMMAND IS GIVEN.

UNIT=42 DSK:/ACCESS=SEQINOU/MOLDE=IMAGE

Enter new file specs.  End with an $(ALT).

*DISPL.SVE$                                     <----
```

Reference: Sections 2.2.1, 2.2.12, 2.2.14.

SCALE

Forces row and column axes to have the same scale on 2D displays in BIPLOT, MULTI, SIDE, SIDE3, and SIDE 6 commands.

```
B:SCALE                                         <----
```

Reference: Display 2.4.

SHIFT

Adjustment of stereo displays. Adjusts horizontal separation of stereo pairs. Set SHIFT equal to zero for analglyphs.

```
B:SHIFT                                         <----

TYPE 1 TO ENTER A NEW SHIFT PARAMETER, OR
     0 TO USE THE DEFAULT.

1                                               <----

ENTER NEW SHIFT PARAMETER FOR STERESCOPIC VIEWS.
SEE MANUAL FOR DETAILS.

.7                                              <----
```

Reference: Sections 2.2.6, 2.2.14.

SIDE

Displays an orthogonal projection of a bimodel onto the observer's viewing plane. Viewing plane can be changed from standard orientation which has the observer at (0, 0,. -3) by use of the ROTATE or the OBSERVER commands. Command requests magnification factor.

```
B:SIDE                                  <----

ENTER MAGNIFICATION FACTOR.

1                                       <----
```

Reference: Sections 2.2.3, 2.2.4, 2.2.14; Display 7.14.

SIDE3

Displays orthogonal projection of a bimodel onto the observer's viewing plane and two orthogonal viewing planes. Viewing plane can be changed from standard orientation by the ROTATE command. (See SIDE.)

```
B:SIDE3                                 <----

ENTER MAGNIFICATION FACTOR.

1                                       <----

TYPE 1 TO FORCE THE SAME SCALES IN ALL 3 VIEWS;
OTHERWISE, TYPE 0.

1                                       <----
```

Reference: Sections 2.2.3, 2.2.4, 2.2.13.

SIDE6

Displays orthogonal projection of a bimodel onto six planes of a rectangular dodecahedron.
(See SIDE and SIDE3.)

```
B:SIDE6                                 <----

ENTER MAGNIFICATION FACTOR.

1                                       <----

TYPE 1 TO FORCE THE SAME SCALES IN ALL 6 VIEWS;
OTHERWISE TYPE 0.

1                                       <----
```

Reference: Sections 2.2.3, 2.2.4, 2.2.13, 7.10a.

STEREO

Displays stereographic pairs of three-dimensional bimodels for stereograms or analglyphs.

```
B:STEREO

TYPE 1 TO PLOT BOTH VIEWS ON ONE PAGE,
     2 TO PLOT THEM SEPARATELY, OR
     3 TO PAUSE BETWEEN VIEWS OF HARD COPY
       ANALYGLYPHS.

1                                        <----
```

Reference: Sections 2.2.3, 2.2.4, 2.2.6; Displays 7.9, 7.15, 7.16.

STOP

Terminates BGRAPH work session and returns control to the monitor. If the work session is to be saved issue a DUMP command.

```
B:STOP                                   <----

STOP

END OF EXECUTION
CPU TIME:  0.82  ELAPSED TIME:  3:4.45
EXIT
```

Reference: Sections 2.2.11, 2.2.14; Display 2.4.

TITLE

Labels graphical output with a title of up to twenty characters.

```
B:TITLE                                  <----

ENTER A TITLE UP TO 20 CHARACTERS LONG:
DISPLAY 31.                              <----
```

Reference: Section 2.2.14; Displays 2.4, 7.4, 7.6.

TRANSFORM

Transforms to a new bimodel factorization via a sequence of rotations, changes of scale, or non-singular linear transformations. After a display command, program prompts for choice of translation (rows or columns) and rotation and scaling. Row and column markers will automatically be transformed inversely to one another. (LAST concatenates these transformations; FIRST resets to identity transformation; default is identity).

B:TRANSFORM <----

THE NEXT 2D OR 3D PLOT COMMAND WILL REQUEST DEFINITION OF A TRANSFORMATION.

Reference: Sections 2.2.10, 2.2.14; Displays 7.5, 7.7.

VIEWPORT

Adjusts stereo displays. Controls size of plot on the display medium. Request for viewport information is made when grahical display commands are called. For stereo displays using standard glasses, use 70% on the Tektronix 4013, or 50% on the Tektronix 4662. A good stereo display can be obtained by using a viewport of 100% and then using a Xerox 9200 copier with a 64% reduction followed by a 74% reduction. VIEWPORT remains in effect until it is called again. The HELP command will report the viewport in effect.

B:VIEWPORT <----

THE NEXT PLOT COMMAND WILL REQUEST VIEWPORT INFORMATION.

Reference: Sections 2.2.6, 2.2.13, 2.2.14; Display 7.2.

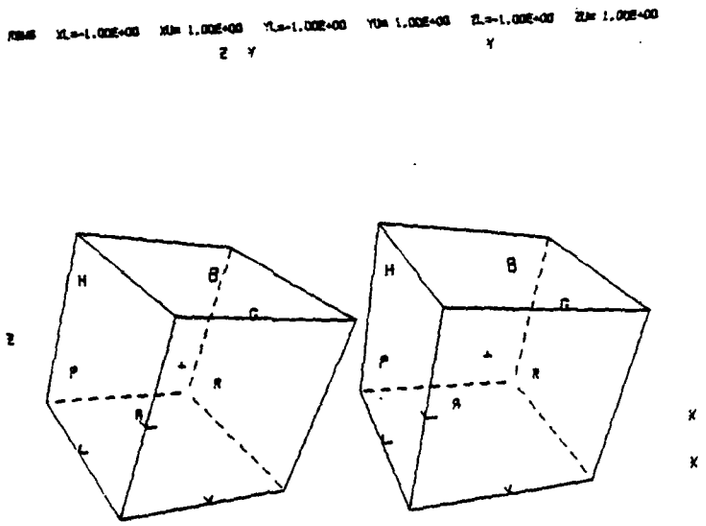
WINDOW

Moves and rescales viewing window of display. WINDOW requests input during the next graphical display command. Remains in effect until *WINDOW* is called again. Edge-lengths less than one zoom into part of the biplot/bimodel; edgelengths greater than one show biplot/bimodel on reduced scale in center of display.

B:WINDOW <----

THE NEXT PLOT COMMAND WILL REQUEST WINDOW INFORMATION.

Reference: Sections 2.2.10, 2.2.13, 2.2.14; Displays 7.2, 7.11.


XL=-1.00E+00 XU= 1.00E+00 YL=-1.00E+00 YU= 1.00E+00 ZL=-1.00E+00 ZU= 1.00E+00
Z Y
Y
Z
H
B
C
P
A
R
Q
L
V
H
B
C
P
A
R
Q
L
V
X
X

7.0 An Illustrated Example of the BGRAPH Commands

7.1 Description of the Example and Data

We use an example extensively analyzed in of Gabriel (1981d, Section 6.5) to demonstrate the features of BGRAPH.

Historical data of annual precipitation in Illinois were used to simulate a weather modification situation. A "cloud seeding" operation was supposed to have taken place in the years 1955-1960 in the Southern Illinois area, and another such operation to have been carried out during 1970-1978 in the Northwestern part of Illinois. No cloud seeding was supposed to have been carried out in Illinois at any other time or place. Central Illinois precipitation could, therefore, serve as concomitant observations to indicate "natural" precipitation; it would not have been "seeded" in any period. (The quotes are used since the data relate to simulated "operations", not to real ones.)

To evaluate the effect of both "operations", it was proposed to use 50 years' data, 1929-1978, for the following five stations: Dubuque and Moline to represent Northwestern Illinois, "seeded" in Period IV -- 1970-1978; St. Louis to represent Southern Illinois, "seeded" in Period II -- 1955-1960; Peoria and Springfield to represent Central Illinois -- never "seeded". These 50 years also provide two "unseeded" periods for comparison, i.e., I -- 1929-1954 and III -- 1961-1969, as set out in Display 7.1a. The corresponding

data for the annual precipitation are shown in Display 7.1b. Note that these are actual precipitation data except that in the "operational" years each "target" station's precipitation was augmented to simulate effects of "seeding".

Display 7.2a shows these data in the form ready for input to program BIPLOT (Gabriel, 1975) including some preliminary parameter values. The rows are individuals years, from 1929 to 1978, the columns are for Dubuque, Moline, Peoria, Springfield and St. Louis. Display 7.2b is the part of program BIPLOT's output that is input into program BGRAPH for display. Display 7.2c is an example of the use of the program SVD to produce a PLQ file for input to BGRAPH.

The Illinois precipitation data may also be analyzed in a MANOVA setting, with the four groups' means at each of the five stations. Display 7.3a shows these means in a format suitable for input to a BIPLOT program, in which they are followed by the "within" matrix of variances and covariances of the five stations. (The first two lines indicate various parameters for MANOVA input including the sample sizes 26, 6, 9 and 9 and $(26 + 6 + 9 + 9 - 4) \times \theta/(1 - \theta) = 23.982$ where $\theta = 0.343$ is the upper five percent point of the maximum characteristic root distribution (Heck, 1960).) The resulting output from program BIPLOT which will be input into BGRAPH is shown in Display 7.3b.

Display 7.1a

Period	No. of Years	Areas and Periods of "Operations" and Comparisons		
		Southern Illinois "Target" (St. Louis)	Northeastern Illinois "Target" (Dubuque, Moline)	Central Illinois Control (Peoria, Springfield)
I. 1929-54	26	Unseeded	Unseeded	Unseeded
II. 1955-60	6	"Seeded"	Unseeded	Unseeded
III. 1961-69	9	Unseeded	Unseeded	Unseeded
IV. 1970-78	9	Unseeded	"Seeded"	Unseeded
Total	50			

Display 7.1b

RAW DATA FILE (LAST2.CLO):

ANNUAL PRECIPIPATION AT 5 ILLINOIS STATIONS 1929-78.

YEAR	DUB	MOL	PEO	SPR	STL
29	24.260	34.710	39.660	33.680	46.300
30	28.350	30.010	24.030	24.320	23.230
31	29.530	31.390	37.750	36.210	37.300
32	25.970	34.490	33.680	32.050	38.010
33	28.700	28.310	34.070	36.470	34.770
34	34.500	36.850	30.430	35.680	29.190
35	32.550	35.580	40.150	41.220	39.360
36	26.770	30.080	30.910	28.920	26.140
37	31.770	30.960	29.890	34.630	35.870
38	47.630	43.750	42.620	36.980	41.220
39	29.890	28.500	38.270	33.050	40.150
40	33.900	25.200	24.160	22.880	25.000
41	32.500	36.940	42.390	44.720	32.120
42	35.570	32.880	37.860	43.360	45.140
43	31.920	32.160	32.810	32.360	33.600
44	42.500	38.930	35.930	33.280	33.510
45	38.780	33.840	36.130	43.400	49.820
46	32.510	38.320	38.890	39.910	57.120
47	42.280	35.630	39.170	36.480	35.760
48	33.350	34.350	30.130	30.860	42.260
49	31.510	34.560	33.330	37.520	45.760
50	32.330	32.880	37.300	32.050	37.630
51	45.010	48.600	37.230	39.510	36.370
52	27.260	28.640	35.430	30.390	25.670
53	34.950	26.470	28.930	23.980	20.590
54	38.210	38.860	41.960	26.670	27.610
55	28.070	26.090	29.990	34.150	40.729
56	24.080	20.200	25.620	31.210	44.759
57	38.920	32.920	36.990	41.970	61.308
58	26.070	24.450	31.450	30.560	48.594
59	54.360	42.100	30.630	35.980	36.803
60	43.360	39.450	37.630	38.910	41.314
61	63.390	45.900	39.450	37.910	41.200
62	42.770	33.850	24.820	30.620	34.610
63	35.440	30.780	25.660	28.890	28.620
64	36.140	35.670	28.950	31.020	32.160
65	61.420	49.590	48.260	39.080	28.260
66	39.230	37.680	33.140	30.700	32.340
67	52.970	42.360	35.950	36.310	41.300
68	39.960	31.850	33.890	31.670	32.490
69	33.700	41.790	33.700	34.680	43.720
70	47.801	67.236	44.720	38.250	36.200
71	48.217	49.972	26.380	27.620	33.730
72	51.714	60.645	36.230	32.030	33.740
73	51.428	73.268	50.220	44.290	39.320
74	50.154	60.879	42.510	40.820	36.830
75	42.263	37.635	41.220	37.660	40.210
76	30.654	32.461	31.230	25.700	23.460
77	50.739	54.548	38.410	42.710	43.410
78	40.300	40.651	32.090	31.830	37.710

Display 7.2a

```
EXAMPLE OF GH BIPLOT TO PRODUCE PLQ FILE FOR INPUT TO SGRAPH:


.COPY FOR01.DAT=ILLIN.CLO

.RUN BIPLOT[5370,5370]
-------------------------------------------------------------------------

INPUT FILE TO BIPLOT PROGRAM (ILLIN.CLO):

GH BIPLOT -- ILLINOIS SIMULATED RAIN FALL DATA
   50     5    1     1     1     2     1     1     0     0
ILLIN.PLQ 0.001     3     1
(5F8.3)
  24.260   34.710   39.660   33.680   46.300
  28.350   30.010   24.030   24.320   23.230
  29.530   31.390   37.750   36.210   37.300
  25.970   34.490   33.680   32.050   38.010
  28.700   28.310   34.070   36.470   34.770
  34.500   36.850   30.430   35.680   29.190
  32.550   35.580   40.150   41.220   39.360
  26.770   30.080   30.910   28.920   26.140
  31.770   30.960   29.890   34.630   35.870
  47.630   43.750   42.620   36.980   41.220
  29.890   28.500   38.270   33.050   40.150
  33.900   25.200   24.160   22.880   25.000
  32.500   36.940   42.390   44.720   32.120
  35.570   32.880   37.860   43.360   45.140
  31.920   32.160   32.810   32.360   33.600
  42.500   38.930   35.930   33.280   33.510
  38.780   33.840   36.130   43.400   49.820
  32.510   38.320   38.890   39.910   57.120
  42.280   35.630   39.170   36.480   35.780
  33.350   34.350   30.130   30.860   42.260
  31.510   34.560   33.330   37.520   45.760
  32.330   32.880   37.300   32.050   37.630
  45.010   48.500   37.230   39.510   36.370
  27.260   28.640   35.430   30.390   25.670
  34.950   26.470   28.830   23.980   20.590
  38.210   38.860   41.960   25.670   27.610
  28.070   26.090   29.990   34.150   40.729
  24.080   20.200   25.620   31.210   44.759
  38.820   32.920   36.990   41.970   61.308
  26.070   24.450   31.450   30.560   48.594
  54.360   42.100   30.630   35.980   36.803
  43.360   39.450   37.630   38.910   41.314
  63.390   45.900   39.450   37.910   41.200
  42.770   33.850   24.820   30.620   34.610
  35.440   30.780   25.660   28.890   28.620
  36.140   35.670   28.950   31.020   32.160
  61.420   49.590   48.250   39.080   28.260
  39.230   37.680   33.140   30.700   32.340
  52.970   42.360   35.950   36.310   41.300
  39.960   31.850   33.390   31.670   32.490
  33.700   41.790   33.700   34.680   43.720
  47.801   67.236   44.720   38.250   36.200
  48.217   49.972   26.380   27.620   33.730
  51.714   60.645   36.230   32.030   33.740
  51.428   73.268   50.220   44.290   39.820
  50.154   60.879   42.510   40.820   36.930
  42.263   37.635   41.220   37.650   40.210
  30.654   32.461   31.230   25.700   23.460
  50.739   54.548   38.410   42.710   43.410
  40.300   40.651   32.090   31.830   37.710
END
```

Display 7.2b

```
OUTPUT FILE (ILLIN.PLQ) FROM BIPLOT TO BE USED AS A PLQ INPUT
FILE TO BGRAPH:

   50     5     3
-.8269D-01 .1933D+00 .2089D+00-.1821D+00-.2318D+00 .3701D-01-.8458D-01 .6824D-01
 .1300D+00-.1038D+00 .4594D-01 .1592D+00-.1239D+00 .3350D-01 .8755D-01-.4749D-01
-.9863D-01 .6445D-01-.1691D-01 .1142D+00 .1478D+00-.1577D+00-.1366D+00 .1459D+00
-.1022D+00 .6747D-02-.9214D-02 .1290D+00 .4788D-01-.4631D-01-.1055D+00 .9416D-01
 .6650D-01-.1846D+00-.2264D+00-.1349D+00-.8796D-03 .4177D-01 .2596D+00-.1410D-01
 .1931D+00-.4508D-02-.9181D-01-.3180D-01 .4772D-01 .2917D-01-.7129D-01-.3643D-01
 .1154D-01 .2374D+00-.1130D+00 .1432D-01 .3375D+00 .1783D-01 .2334D-01-.2623D-02
-.3384D-01-.6990D-01 .5799D-01-.6260D-01-.5144D-01 .1628D+00 .8569D-02-.6795D-01
 .3224D-01 .7132D-01 .1331D+00-.2043D-01 .3275D-01-.1493D+00-.1193D+00 .1868D+00
-.1582D+00-.2712D+00-.4429D-01-.7762D-04-.1565D+00 .1438D+00-.1541D+00 .9432D-01
-.2970D-01-.2341D+00 .1446D+00-.1132D+00 .1657D-01 .3892D+00-.2076D+00-.1730D+00
 .1947D+00-.5471D-01 .1154D+00-.7776D-01-.3091D+00 .6354D-01 .7018D-01-.6867D-01
 .2296D+00-.1984D-01-.3559D+00-.4203D-01-.9805D-01-.2519D+00-.1145D+00-.1535D+00
-.9486D-01-.5780D-01-.9507D-01-.3593D-01 .2586D+00-.1609D+00-.6386D-01-.1389D-01
-.9703D-01-.2254D-01 .1299D+00 .5836D-02-.2477D+00-.4645D-01-.7676D-01-.8637D-01
 .5228D-02 .9516D-01 .5520D-01 .3001D+00-.5723D-01 .2584D+00 .1010D+00-.1734D+00
-.1673D+00 .2354D+00-.1560D+00 .1779D-01 .3973D+00 .1390D-01 .3243D+00 .2639D+00
-.3496D-01 .1328D+00 .5063D-01 .6593D-01-.1758D-01-.1272D+00-.2112D+00 .1129D+00
 .2258D+00 .6437D-01-.4201D-01 .1856D-01-.3096D-01-.6295D-01
 .5894D+00-.2342D+00-.7124D+00 .7123D+00-.1368D+00 .4008D+00 .2893D+00 .2015D+00
 .4768D+00 .2246D+00 .3868D+00 .1241D+00 .1052D+00 .8581D+00-.2984D+00
 .1005D+03 .6322D+02 .3797D+02
```

Display 7.2c

```
EXAMPLE OF USE OF SVD PROGRAM TO PRODUCE PLQ FILE FOR INPUT TO BGRAPH:


.RUN SVD[5370,5370]

ENTER # ROWS, # COLUMNS AND # COMPONENTS:50 5 3    <----


TYPE 1 TO ENTER MATRIX FROM TTY; OTHERWISE
ENTER FILENAME:LAST2.CLO                           <----


TYPE * TO READ IN FREE FORMAT OR F TO ENTER A
FORMAT(MATRIX WILL BE READ ROW BY ROW):*           <----

TYPE 0 TO USE RAW DATA,
     1 TO USE DEVIATIONS FROM ROW MEANS,
     2 TO USE DEVIATIONS FROM COLUMN MEANS,
     3 TO USE DEVIATIONS FROM THE GRAND MEAN, OR
     4 TO USE RESIDUALS FROM ADDITIVITY.

2                                                  <----

MATRIX NORM=    0.167983E+05

                          SS              %
COMPONENT  1      0.101003E+05      0.601268E+02
REMAINING         0.669799E+04      0.398732E+02

                          SS              %
COMPONENT  2      0.399669E+04      0.237923E+02
REMAINING         0.270130E+04      0.160808E+02

                          SS              %
COMPONENT  3      0.144188E+04      0.858352E+01
REMAINING         0.125942E+04      0.749732E+01

ENTER FILENAME WHERE THE SVD IS TO BE STORED:SVD.PLQ    <----


STOP
```

DISPLAY 7.3a: EXAMPLE OF MANOVA BIPLOT TO PRODUCE PLQ FILE FOR INPUT TO BGRAPH.

```
.COPY FOR01.DAT=MANOVA.CLO

.RUN BIPLOT[5370,5370]

STOP

END OF EXECUTION
CPU TIME: 3.76  ELAPSED TIME: 9.25
EXIT
```

INPUT FILE (MANOVA.CLO) TO BIPLOT PROGRAM TO DO MANOVA BIPLOT:

```
MANOVA BIPLOT -- ILLINOIS SIMULATED RAIN FALL DATA
    4     5     1     1     1     3     2     5     1     1
MANOV.PLQ 0.001     3     1   23.8920   0.00000   0.00000
 (5F15.5)
       33.55769       33.95731       35.11577       34.25308       36.13538
       35.79333       30.86833       32.05167       35.46333       45.58450
       45.00222       38.83000       33.75778       33.43111       34.96667
       45.91889       53.03278       38.11222       35.65667       36.12333
      26.       6.       9.       9.
(5F8.3)
57.537  29.067  22.310  15.094   5.330
29.067  30.704  21.439  16.830  15.638
22.310  21.439  32.442  20.165  16.970
15.094  16.830  20.165  29.909  28.789
 5.330  15.638  16.970  28.789  65.383
END
```

DISPLAY 7.3b: OUTPUT PLQ FILE (MANOV.PLQ) FROM MANOVA BIPLOT FOR INPUT TO BGRAPH.

```
    4    5    3
 .6520D-01 .1163D+00-.2626D-01 .1578D+00-.1952D+00 .2893D+00-.6876D-02-.2158D+00
-.2110D+00-.2867D+00 .9958D-02 .9397D-01
-.2418D+01-.4624D+01-.2740D+01-.4159D+01-.8058D+00 .5630D+00-.8365D+00 .1265D+01
-.7630D-01-.1923D+00 .5942D-01 .1797D+01 .7177D+00-.1713D+01 .6896D+01
 .1261D+02 .5133D+01 .2698D+01
```

7.2 Examples of Commands Used and Display Obtained

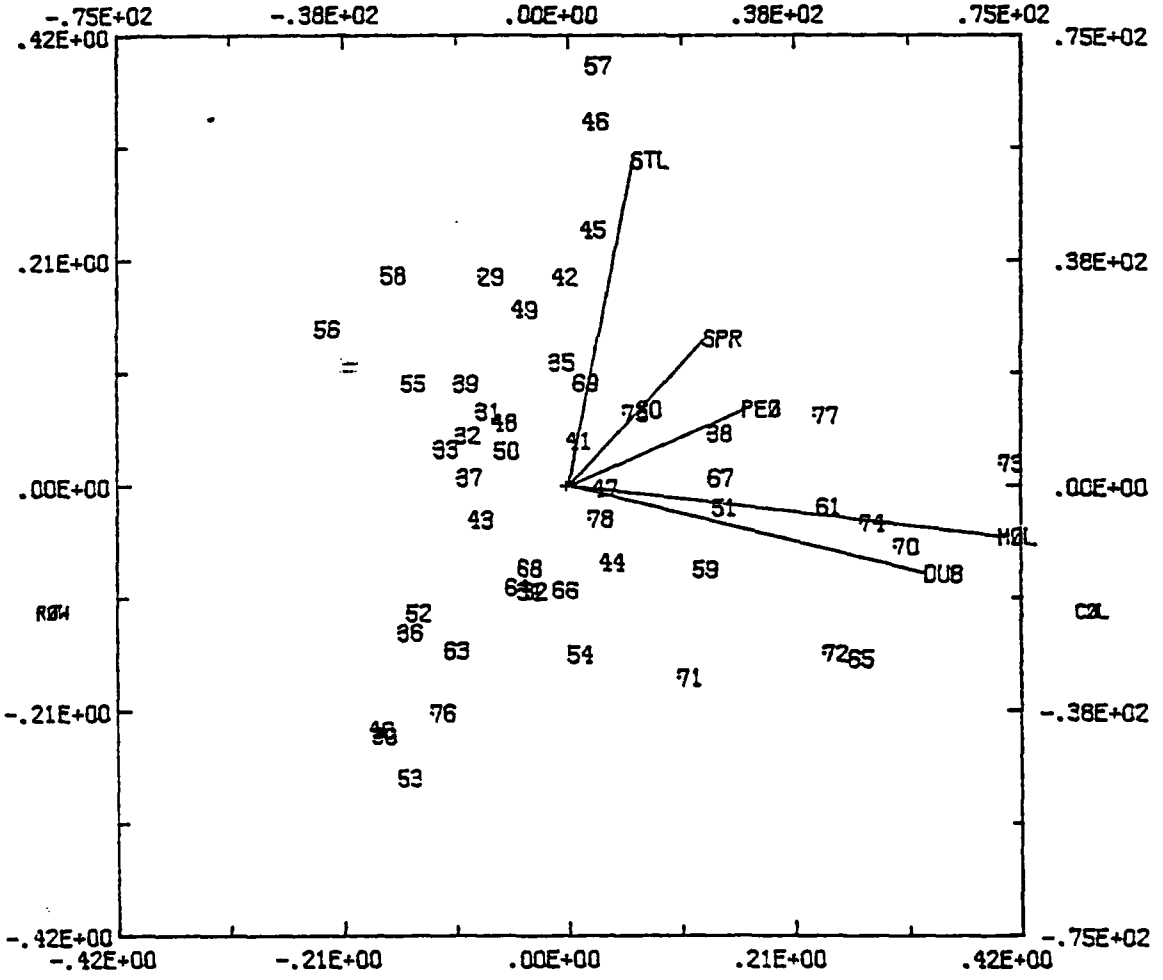
The Illinois rainfall data were inspected with the help of a number of biplot/bimodel displays. In the following, we present side-by-side a number of such displays (chosen to illustrate BGRAPH capabilities) and the BGRAPH commands used to generate each of them. Since many BGRAPH options carry over from one display to the next -- and may need to be explicitly reset -- it is important to note that these displays were generated sequentially, with each set of commands actually following upon the previous commands and displays --exactly as illustrated here. (For further discussion of this example, see Gabriel, 1981d.)

The following examples are reduced Xerox copies from plots produced by the Tektronix 4662 plotter. The actual size of the plots can be controlled by VIEWPORT or WINDOW commands (and magnification prompt). On the CRT, the size of the display will be correspondingly smaller.

Display 7.4a: READ, TITLE, CVECTOR, CLABEL, RLABEL, BIPLOT.

```
RU BGRAPH

ENTER TERMINAL TYPE CODE
0-DEC 340
1-COMPUTEK 400
2-ADAGE AGT-30
3-ARDS
4-TEKTRONIX 4010,4013
5-DEC GT40
6-TEKTRONIX 4014
7-OTHER
7


B:READ
ENTER NAME OF PLQ FILE:
Unit=41   :/ACCESS=SEQINOU/MODE=ASCII

Enter new file specs. End with an $(ALT)
*LAST2.PLQ$

   3 COMPONENTS READ.
ENTER: 1 FOR GH FACTORIZATION,
       2 FOR JK FACTORIZATION,
       3 FOR GF FACTORIZATION,
       4 FOR EK FACTORIZATION,
       5 FOR SQRT FACTORIZATION.
1
B:TITLE
ENTER A TITLE UP TO 20 CHARACTERS LONG: ILLINOIS RAINFALL
B:CVECTOR
COLUMN MARKERS WILL BE PLOTTED AS VECTORS.
B:CLABEL
CHOOSE LABELS FOR COLUMN MARKERS BY ENTERING:
1 FOR COLUMN NUMBER(PREFIXED BY C),
2 FOR RANKS ON SOME VARIABLE,
3 FOR DECILE RANKS MINUS 1(I.E. 0 TO 9),
4 FOR STANDARD SCORES ON SOME VARIABLE,
5 TO ENTER LABELS(MAX. LENGTH=5 CHARACTERS).
5
ENTER 1 IF LABELS OR LABELING VARIABLE ARE TO BE
I TERED FROM THE TTY, OTHERWISE ENTER 0.
1
ENTER FORMAT FOR LABELS OR LABELING VARIABLE.
USE A-FORMAT)
(A3)
ENTER LABELING VARIABLE OR LABELS(    5 VALUES OR
LABELS IN ALL):
DUB
MOL
PEO
SPR
STL
B:RLABEL
CHOOSE LABELS FOR ROW MARKERS BY ENTERING:

1 FOR ROW NUMBER,
2 FOR RANKS ON SOME VARIABLE,
3 FOR DECILE RANKS MINUS 1(I.E. 0 TO 9),
4 FOR STANDARD SCORES ON SOME VARIABLE,
5 TO ENTER LABELS(MAX. LENGTH=5 CHARACTERS).
5
ENTER 1 IF LABELS OR LABELING VARIABLE ARE TO BE
ENTERED FROM THE TTY, OTHERWISE ENTER 0.
1
ENTER FORMAT FOR LABELS OR LABELING VARIABLE.
USE A-FORMAT)
(10A3)
ENTER LABELING VARIABLE OR LABELS(   50 VALUES OR
LABELS IN ALL):
29 30 31 32 33 34 35 36 37 38
39 40 41 42 43 44 45 46 47 48
49 50 51 52 53 54 55 56 57 58
59 60 61 62 63 64 65 66 67 68
69 70 71 72 73 74 75 76 77 78
B:BIPLOT   (Plot 7.4)
```

Display 7.4b: READ, TITLE, CVECTOR, CLABEL, RLABEL, BIPLOT.



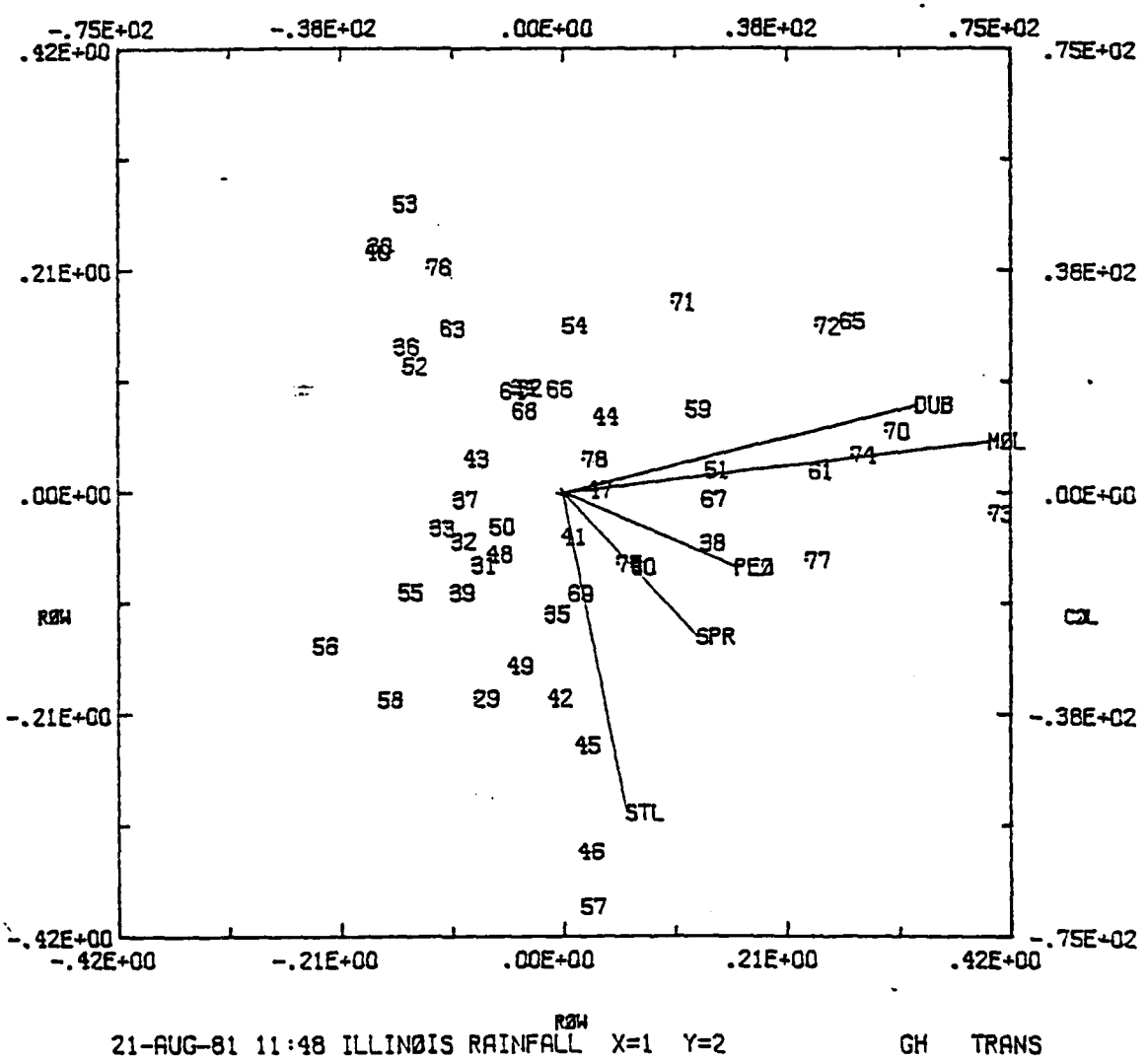
RØW

21-AUG-81 11:47 ILLINØIS RAINFALL X=1 Y=2 GH

Display 7.5a: TRANSFORM, BIPLOT. Use scale factor to reflect the biplot about the x-axis.

```
B:TRANSFORM
THE NEXT 2D OR 3D PLOT COMMAND WILL REQUEST DEFINITION
OF A TRANSFORMATION.
B:BIPLOT    (Plot 7.5)
DO YOU WISH TO TRANSLATE ROW  MARKERS(Y OR N)?:N
DO YOU WISH TO TRANSLATE COL. MARKERS(Y OR N)?:N
TYPE 1 TO ROTATE ABOUT (NEW) ORIGIN
     2 TO SCALE
     3 FOR A GENERAL LINEAR TRANFORMATION, OR
     4 TO END
2
TYPE R TO ENTER SCALE FACTORS FOR ROW MARKERS, OR C
FOR COLUMN MARKERS. WHICHEVER IS ENTERED, THE OTHER
MARKERS WILL BE RECIPROCALLY SCALED.
R
ENTER SCALE FACTORS:
1 -1
TYPE 1 TO ROTATE ABOUT (NEW) ORIGIN
     2 TO SCALE
     3 FOR A GENERAL LINEAR TRANFORMATION, OR
     4 TO END
4
```

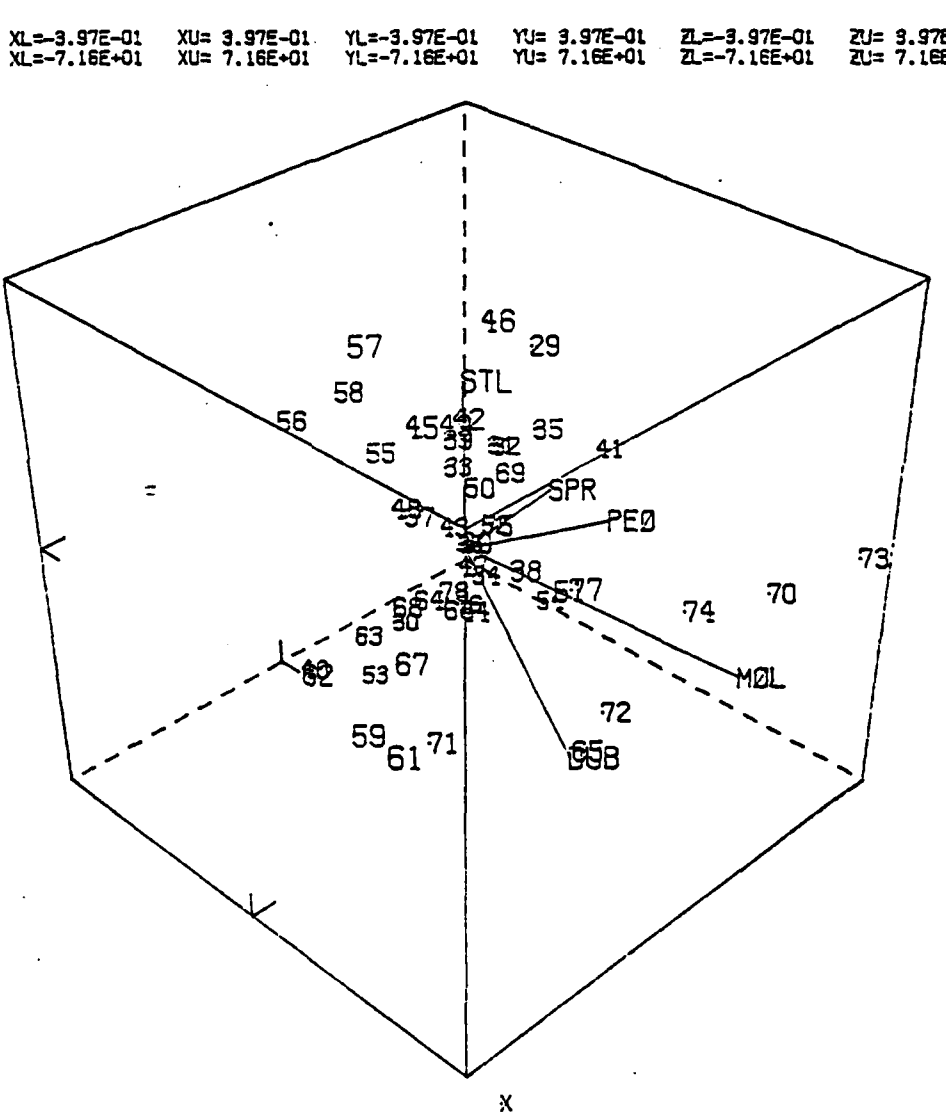
Display 7.5b: TRANSFORM, BIPLOT.



Display 7.6a: TITLE, OBSERVER, ROTATE, PERSPECTIVE. Change position of observer and rotate cube clockwise 45°.

```
B:TITLE
ENTER A TITLE UP TO 20 CHARACTERS LONG:
0
B:OBSERVER
TYPE 1 TO ENTER AN OBSERVATION POINT, OR
     0 TO USE THE DEFAULT.
1
ENTER COORDINATES OF THE NEW OBSERVATION POINT(MUST BE AT LEAST
SQRT(3) CUBE UNITS FROM THE ORIGIN.
0 2 -3
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
1
ENTER 3 ANGLES(IN DEGREES)TO ROTATE THE CUBE COUNTER-
CLOCKWISE ABOUT AXES THROUGH THE CENTER OF THE CUBE,
FIRST ABOUT THE X-AXIS,THEN THE Y-AXIS AND FINALLY THE Z-AXIS.
0 -45 0
B:PERSP    (Plot 7.6)
```

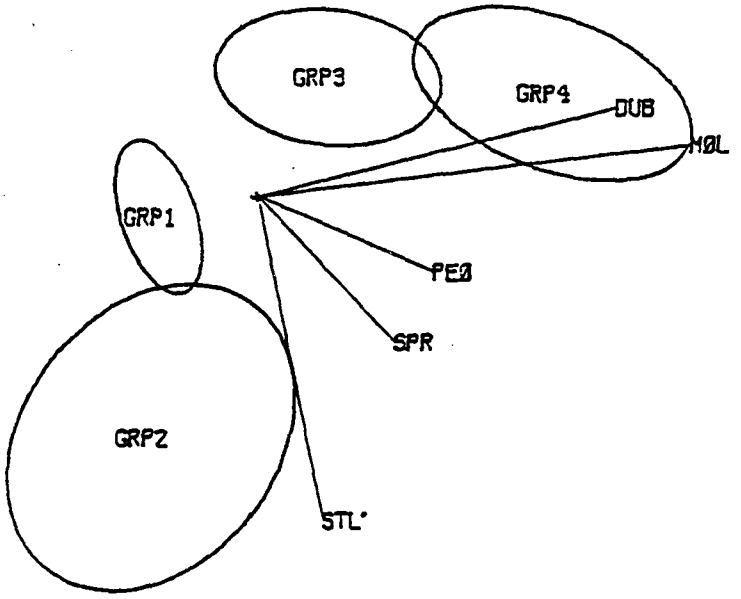
Display 7.6b: TITLE, OBSERVER, ROTATE, PERSPECTIVE.



Display 7.7a: AXES, TRANSFORM, ELLIPSE, BIPLOT. Suppress axes, reflect biplot about x-axis, and display ellipsoids of concentration with labels.

```
B:AXES

TYPE 0 TO SUPPRESS BIPLOT AXES, OTHERWISE TYPE 1
0
B:TRANSFORM
THE NEXT 2D OR 3D PLOT COMMAND WILL REQUEST DEFINITION
OF A TRANSFORMATION.
B:ELLIPSE
TYPE 1 TO DRAW CONCENTRATION ELLIPSOIDS, OR 0 TO OMIT THEM.
1
TYPE 1 TO PLOT ROW MARKERS AS WELL,
     2 TO EXCLUDE MARKERS INSIDE THEIR RESPECTIVE ELLIPSOIDS, OR
     3 TO SUPPRESS PLOTTING OF ALL ROW MARKERS.
3
B:BIPLOT   (Plot 7.4)
DO YOU WISH TO TRANSLATE ROW  MARKERS(Y OR N)?:N
DO YOU WISH TO TRANSLATE COL. MARKERS(Y OR N)?:N
TYPE 1 TO ROTATE ABOUT (NEW) ORIGIN
     2 TO SCALE
     3 FOR A GENERAL LINEAR TRANFORMATION, OR
     4 TO END
2
TYPE R TO ENTER SCALE FACTORS FOR ROW MARKERS, OR C
FOR COLUMN MARKERS. WHICHEVER IS ENTERED, THE OTHER
MARKERS WILL BE RECIPROCALLY SCALED.
R
ENTER SCALE FACTORS:
1 -1
TYPE 1 TO ROTATE ABOUT (NEW) ORIGIN
     2 TO SCALE
     3 FOR A GENERAL LINEAR TRANFORMATION, OR
     4 TO END
4
TYPE R TO REUSE PREVIOUSLY ENTERED GROUP SPECIFICATIONS,
     E TO ENTER NEW SPECIFICATIONS, OR
     A TO ABORT THE ELLIPSE OPTION.
E
ENTER NUMBER OF GROUPS(MAX.=10):4
ENTER   4 GROUP LABELS IN 10A5 FORMAT.
GRP1 GRP2 GRP3 GRP4
GROUP CODES ARE INTEGERS FROM 0 TO  4 WITH 0 INDICATING
EXCLUSION FROM ALL GROUPS.   50 GROUP CODES MUST BE ENTERED.
TYPE 1 TO ENTER CODES FROM TTY,OTHERWISE TYPE 0.
1
ENTER (INTEGER) FORMAT FOR READING CODES.
(10I1)
ENTER GROUP CODES:
1111111111
1111111111
1111112222
2233333333
3444444444
ENTER POSITIVE VALUE INDICATING NUMBER OF STANDARD DEVIATIONS
OR STANDARD ERRORS INSIDE ELLIPSOIDS.
1
TYPE 1 FOR STANDARD DEVIATIONS, OR 0 FOR STANDARD ERRORS.
1
```

Display 7.7b: AXES, TRANSFORM, ELLIPSE, BIPLOT.

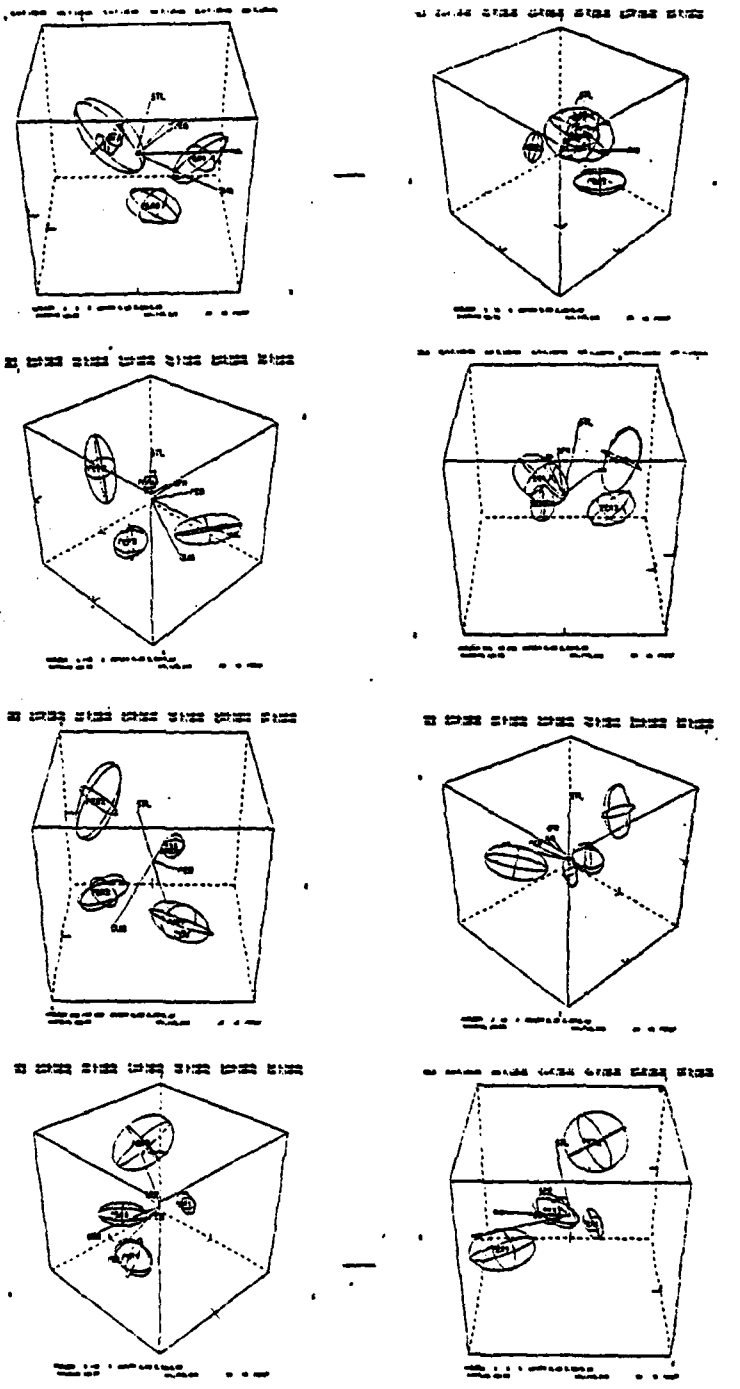


23-AUG-81 15:29 X=1 Y=2 GH TRANS

Display 7.8a: OBSERVER, LAST, ROTATE, PERSPECTIVE. Perspective plots with eight successive 45 degree rotations about the Y-axis. Observer at (0.0, 2.0, -3.0).

```
B:OBSERVER
TYPE 1 TO ENTER AN OBSERVATION POINT, OR
     0 TO USE THE DEFAULT.
1
ENTER COORDINATES OF THE NEW OBSERVATION POINT(MUST BE AT LEAST
SQRT(3) CUBE UNITS FROM THE ORIGIN.
0.0 2.0 -3.0
B:LAST
TRANSFORMATIONS WILL BE CONCATENATED.
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
2
TO ROTATE THE CUBE COUNTER-CLOCKWISE ABOUT AN AXIS THROUGH
THE CENTER OF THE CUBE, ENTER A VECTOR TO DEFINE THE AXIS
AND AN ANGLE(IN DEGREES).
0 1 0 45
B:PERSPECTIVE
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
2
TO ROTATE THE CUBE COUNTER-CLOCKWISE ABOUT AN AXIS THROUGH
THE CENTER OF THE CUBE, ENTER A VECTOR TO DEFINE THE AXIS
AND AN ANGLE(IN DEGREES).
0 1 0 45
B:PERSPECTIVE
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
2
TO ROTATE THE CUBE COUNTER-CLOCKWISE ABOUT AN AXIS THROUGH
THE CENTER OF THE CUBE, ENTER A VECTOR TO DEFINE THE AXIS
AND AN ANGLE(IN DEGREES).
0 1 0 45
B:PERSPECTIVE
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
2
TO ROTATE THE CUBE COUNTER-CLOCKWISE ABOUT AN AXIS THROUGH
THE CENTER OF THE CUBE, ENTER A VECTOR TO DEFINE THE AXIS
AND AN ANGLE(IN DEGREES).
0 1 0 45
B:PERSPECTIVE
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
2
TO ROTATE THE CUBE COUNTER-CLOCKWISE ABOUT AN AXIS THROUGH
THE CENTER OF THE CUBE, ENTER A VECTOR TO DEFINE THE AXIS
AND AN ANGLE(IN DEGREES).
0 1 0 45
B:PERSPECTIVE
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
2
TO ROTATE THE CUBE COUNTER-CLOCKWISE ABOUT AN AXIS THROUGH
THE CENTER OF THE CUBE, ENTER A VECTOR TO DEFINE THE AXIS
AND AN ANGLE(IN DEGREES).
0 1 0 45
B:PERSPECTIVE

B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
2
TO ROTATE THE CUBE COUNTER-CLOCKWISE ABOUT AN AXIS THROUGH
THE CENTER OF THE CUBE, ENTER A VECTOR TO DEFINE THE AXIS
AND AN ANGLE(IN DEGREES).
0 1 0 45
B:PERSPECTIVE
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
2
TO ROTATE THE CUBE COUNTER-CLOCKWISE ABOUT AN AXIS THROUGH
THE CENTER OF THE CUBE, ENTER A VECTOR TO DEFINE THE AXIS
AND AN ANGLE(IN DEGREES).
0 1 0 45
B:PERSPECTIVE
```

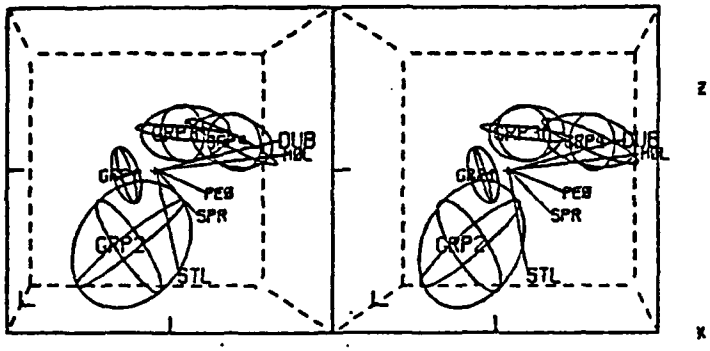
Display 7.8b: OBSERVER, ROTATE, PERSPECTIVE

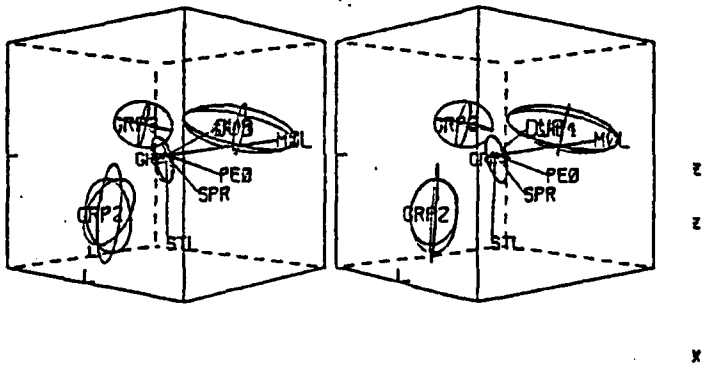


Display 7.9a: OBSERVER, ROATE, STEREO. Rotated stereo display with observer position returned to default.

```
B:OBSERV
TYPE 1 TO ENTER AN OBSERVATION POINT, OR
     0 TO USE THE DEFAULT.
0
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
1
ENTER 3 ANGLES(IN DEGREES)TO ROTATE THE CUBE COUNTER-
CLOCKWISE ABOUT AXES THROUGH THE CENTER OF THE CUBE,
FIRST ABOUT THE X-AXIS,THEN THE Y-AXIS AND FINALLY THE Z-AXIS.
0 -45 0
B:STERE    (Plot 7.6)
TYPE 1 TO PLOT BOTH VIEWS ON 1 PAGE,
     2 TO PLOT THEM SEPARATELY, OR
     3 TO PAUSE BETWEEN VIEWS OF HARD COPY ANALGLYPHS.
1
TYPE R TO REUSE PREVIOUSLY ENTERED GROUP SPECIFICATIONS,
     E TO ENTER NEW SPECIFICATIONS, OR
     A TO ABORT THE ELLIPSE OPTION.
R
ENTER POSITIVE VALUE INDICATING NUMBER OF STANDARD DEVIATIONS
OR STANDARD ERRORS INSIDE ELLIPSOIDS.
1
TYPE 1 FOR STANDARD DEVIATIONS, OR 0 FOR STANDARD ERRORS.
0
```

Display 7.9b: OBSERVER, ROTATE, STEREO. (Front view of stereogram shown for completeness.)

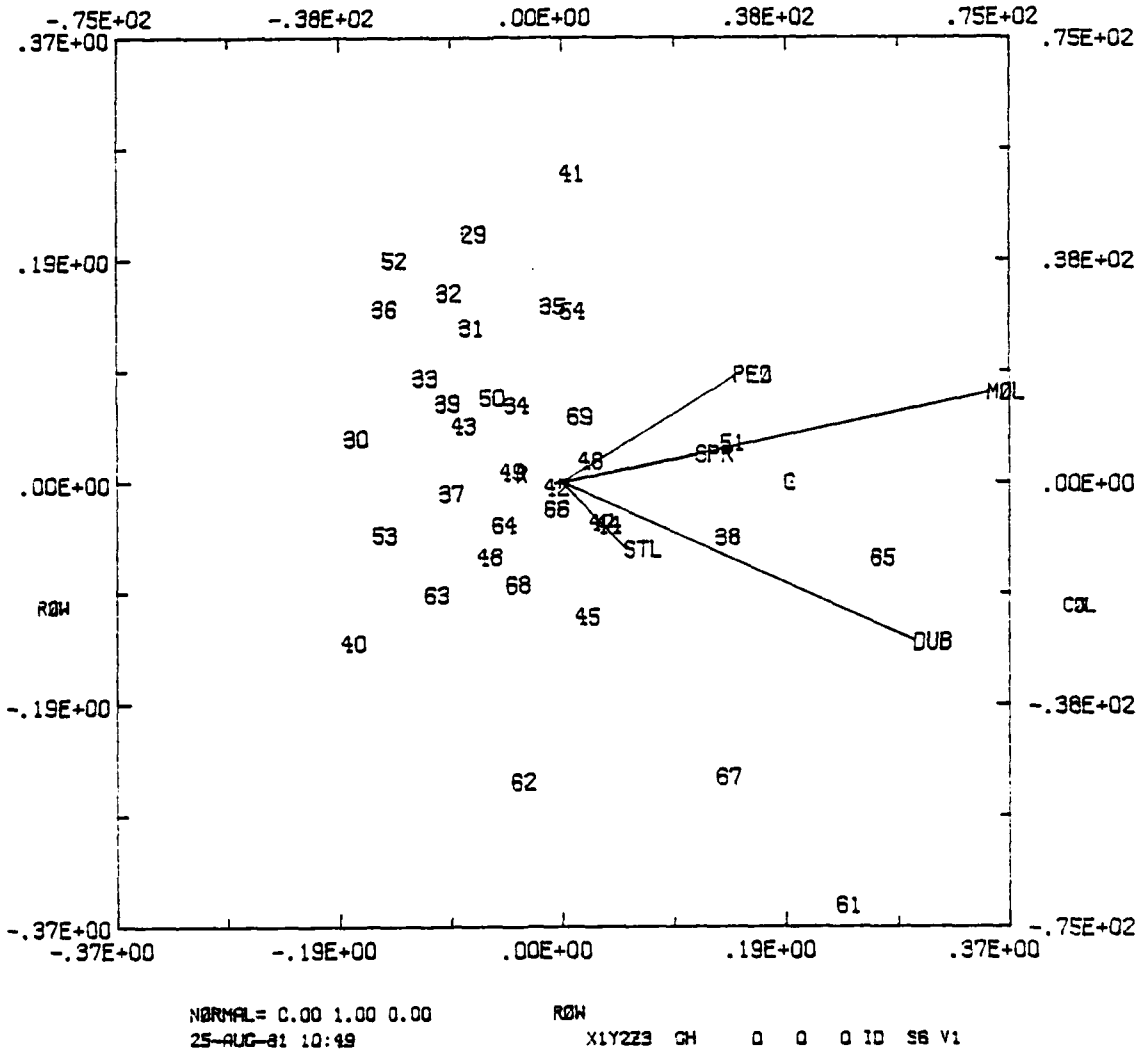




Display 7.10a: RPICK, MEANS, SIDE6. Select a subset of rows, and display biplot on six sides of a dodecahedron.

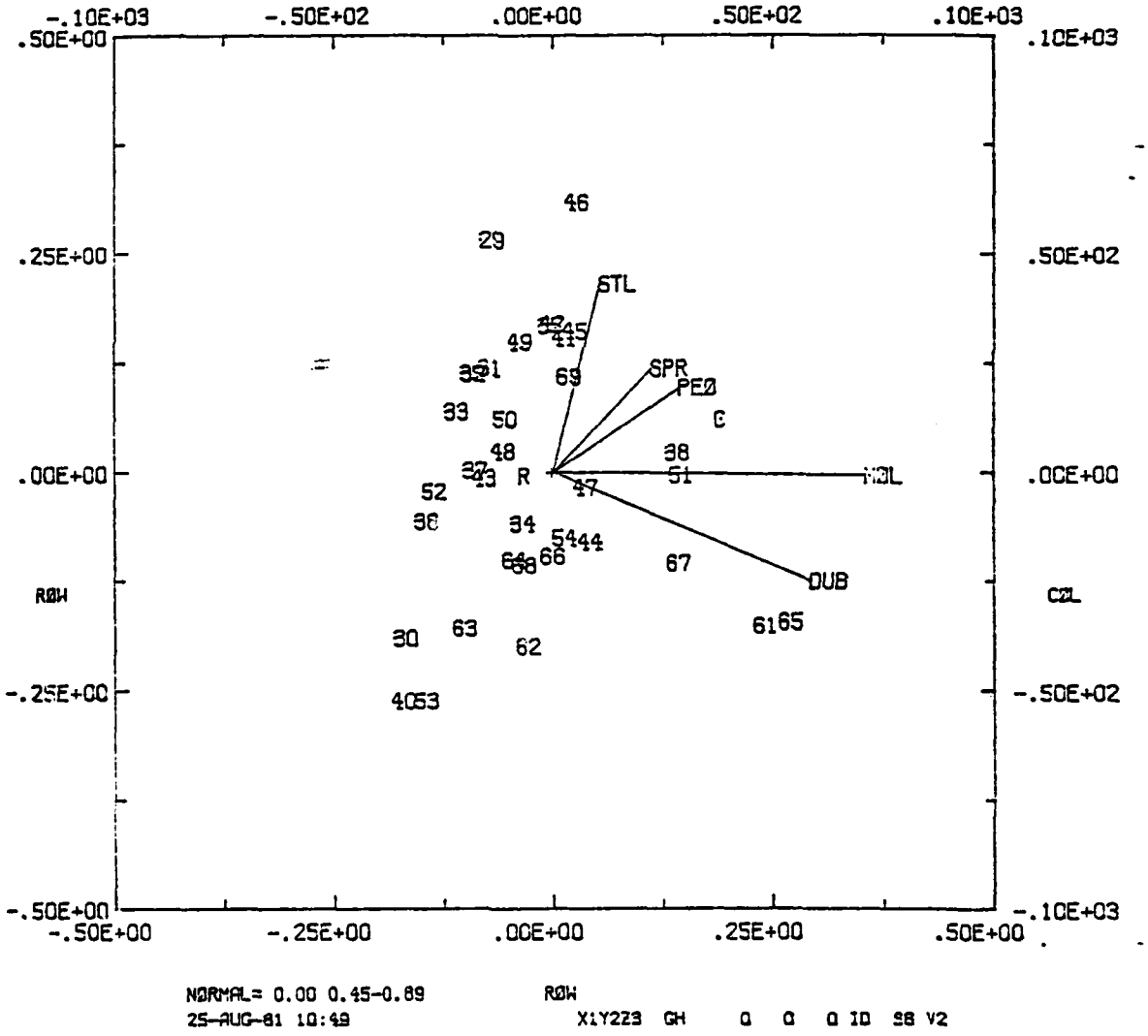
```
B:RPICK
ENTER LIST OF ROW NUMBERS TO BE INCLUDED(3-5=3,4,5).
END WITH A / .
1-26 33-41 /
B:MEANS
TYPE 1 TO PLOT MARKER MEANS, OR
     0 TO OMIT THEM.
1
B:SIDE6    (Plots 7.7.1-7.7.6)

TYPE R TO REUSE PREVIOUSLY ENTERED GROUP SPECIFICATIONS,
     E TO ENTER NEW SPECIFICATIONS, OR
     A TO ABORT THE ELLIPSE OPTION.
A
ENTER MAGNIFICATION FACTOR.
1
TYPE 1 TO FORCE THE SAME SCALES IN ALL 6 VIEWS
OTHERWISE TYPE 0.
0
```

Display 7.10b: RPICK, MEANS, SIDE6 (View 1).

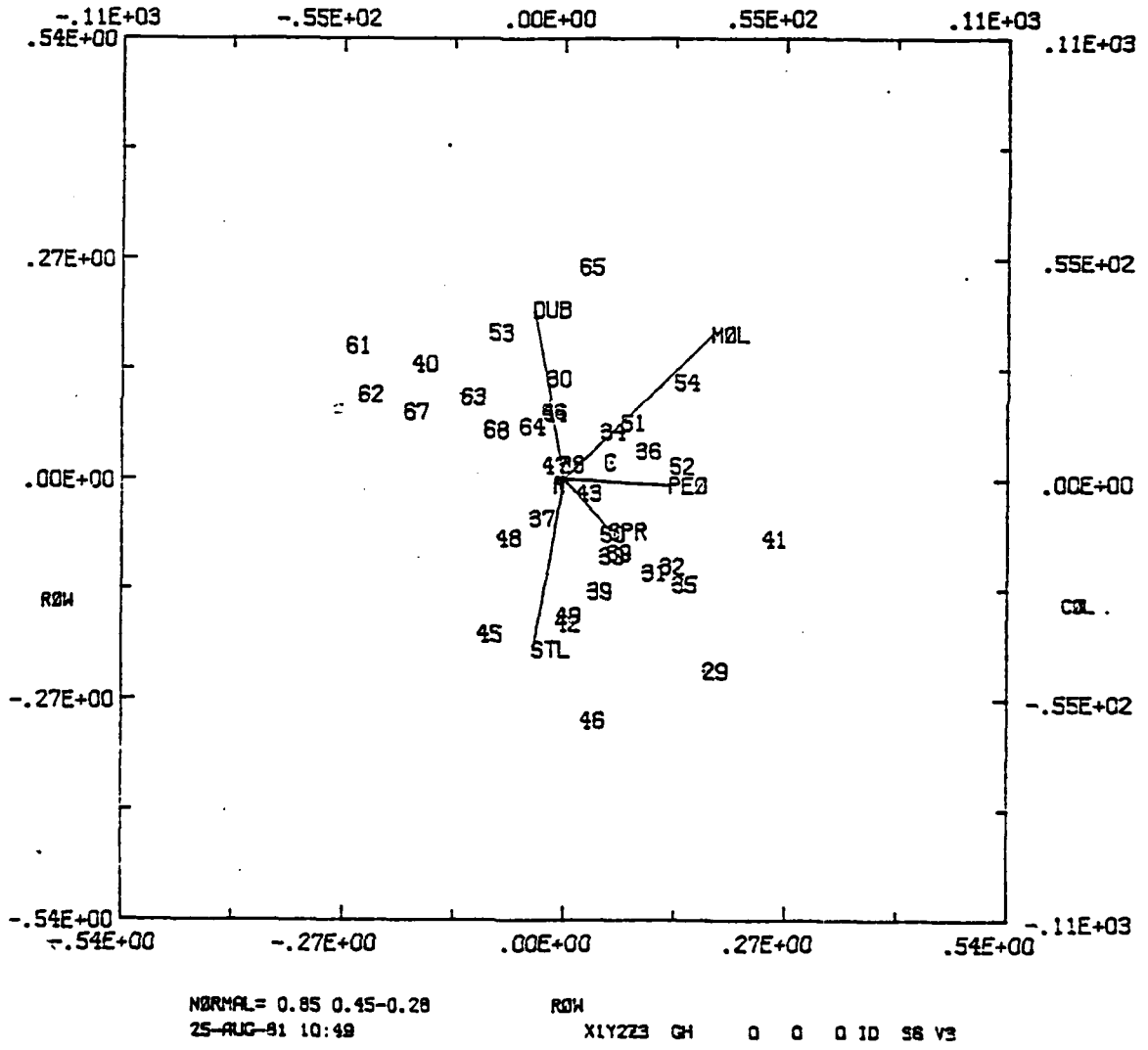


Note: NORMAL gives coordinates of a vector normal to the plot.

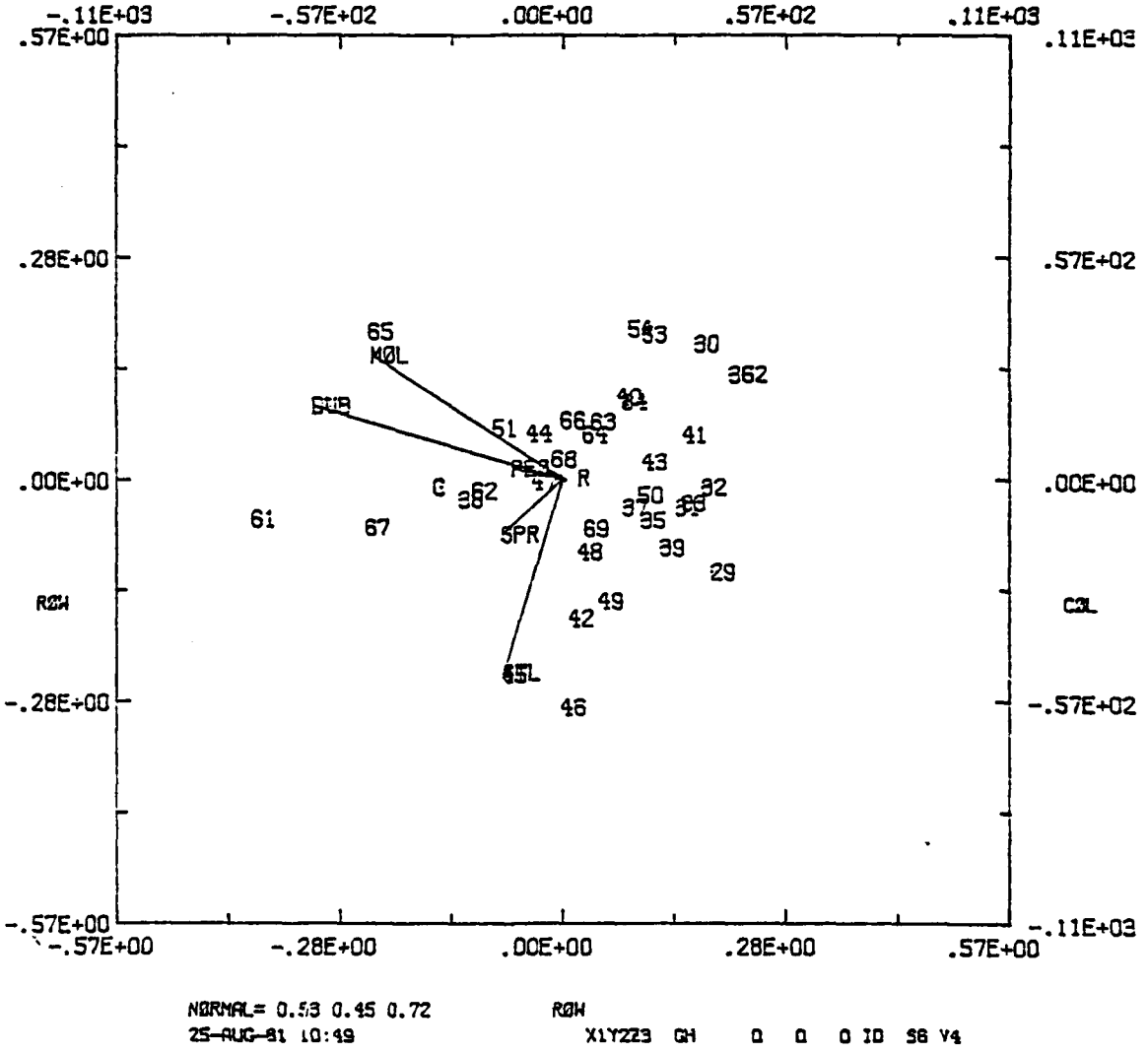
Display 7.10c: RPICK, MEANS, SIDE6 (View 2).



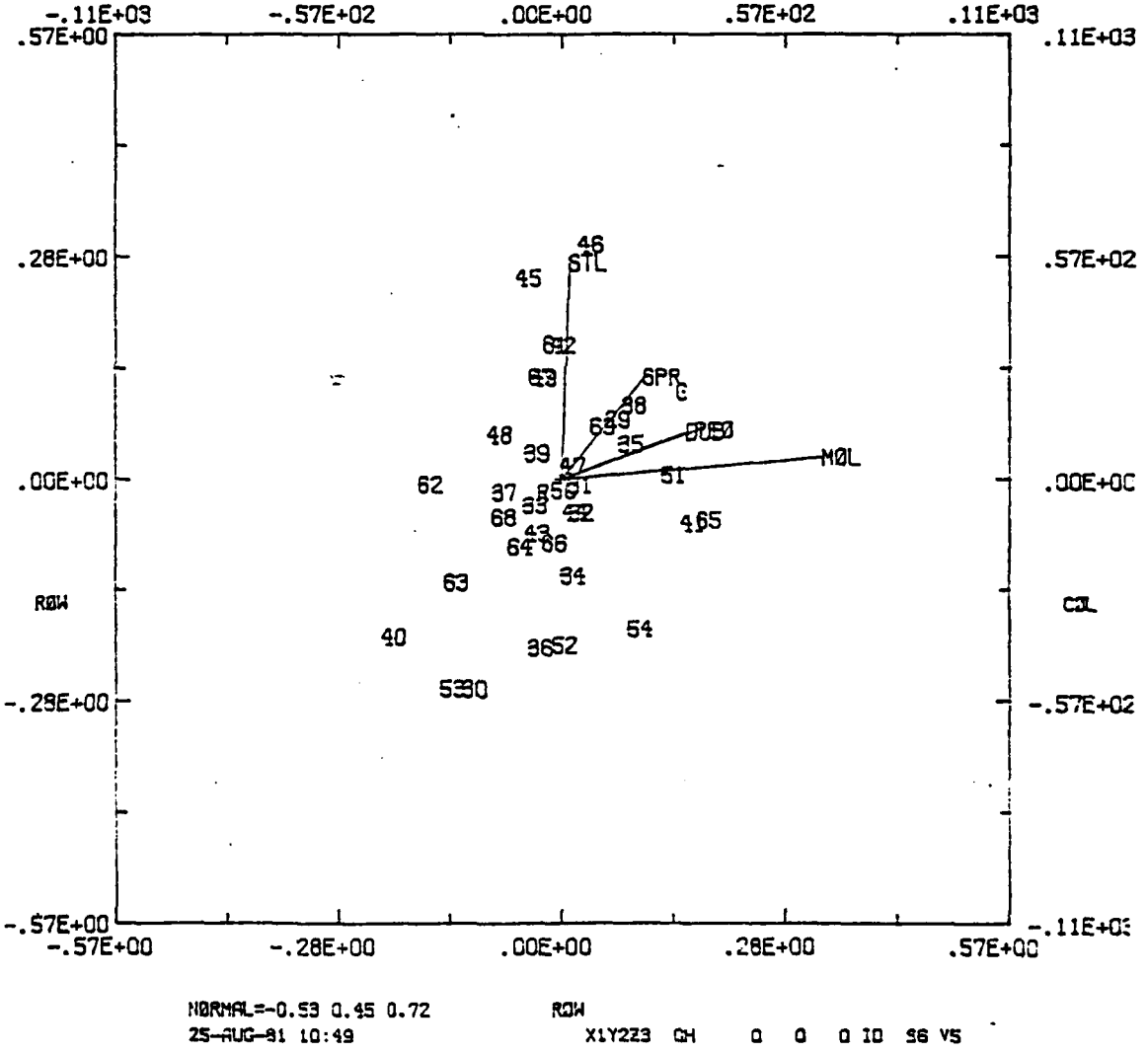
Display 7.10d: RPICK, MEANS, SIDE6 (View 3).



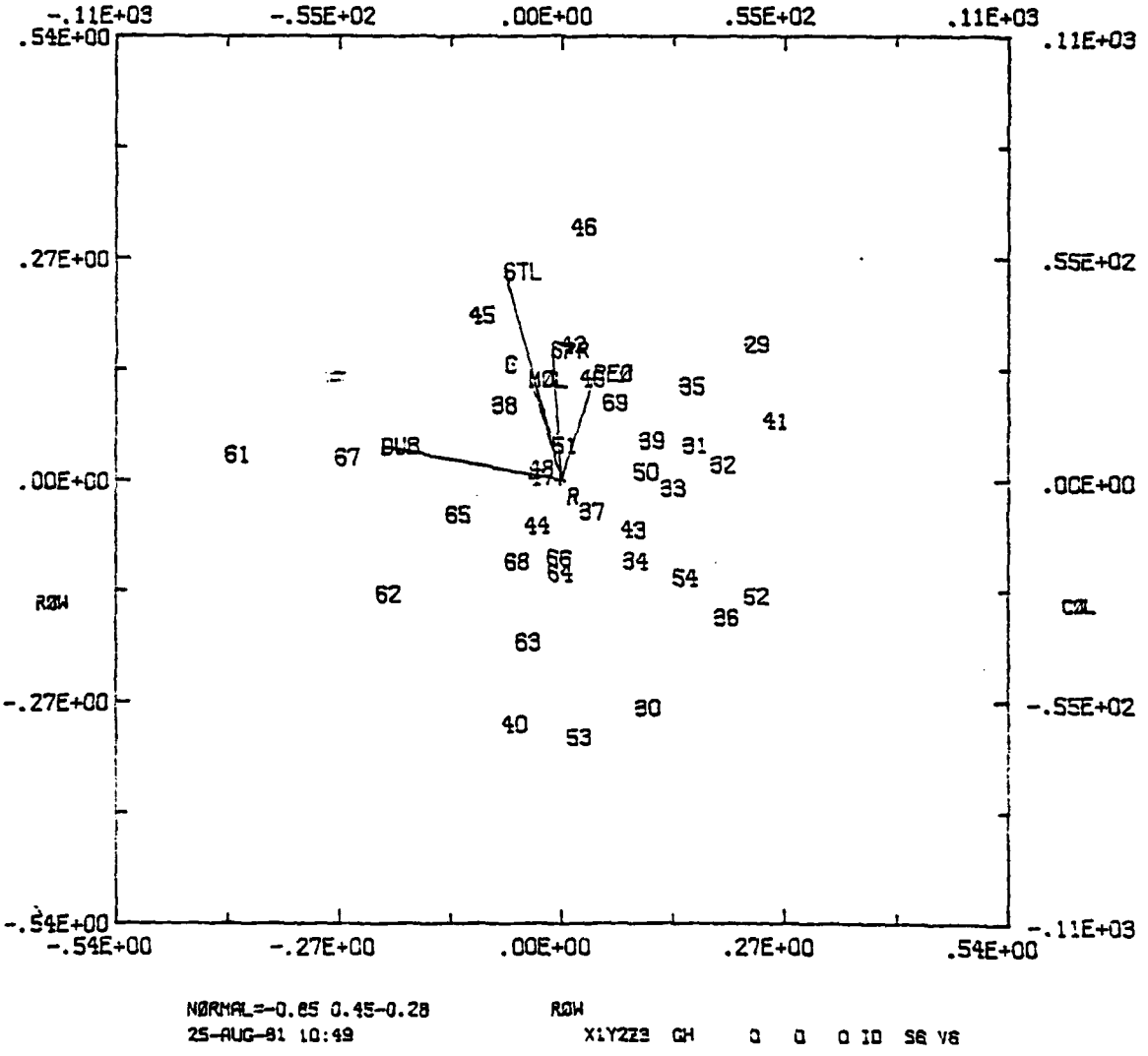
Display 7.10e: RPICK, MEANS, SIDE6 (View 4).

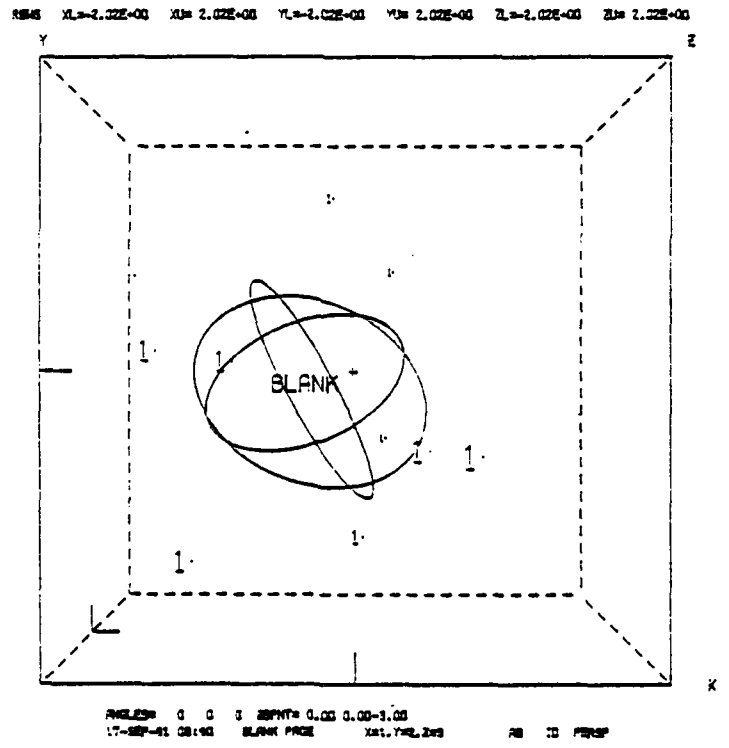


Display 7.10f: RPICK, MEANS, SIDE6 (View 5).



Display 7.10g: RPICK, MEANS, SIDE6 (View 6).

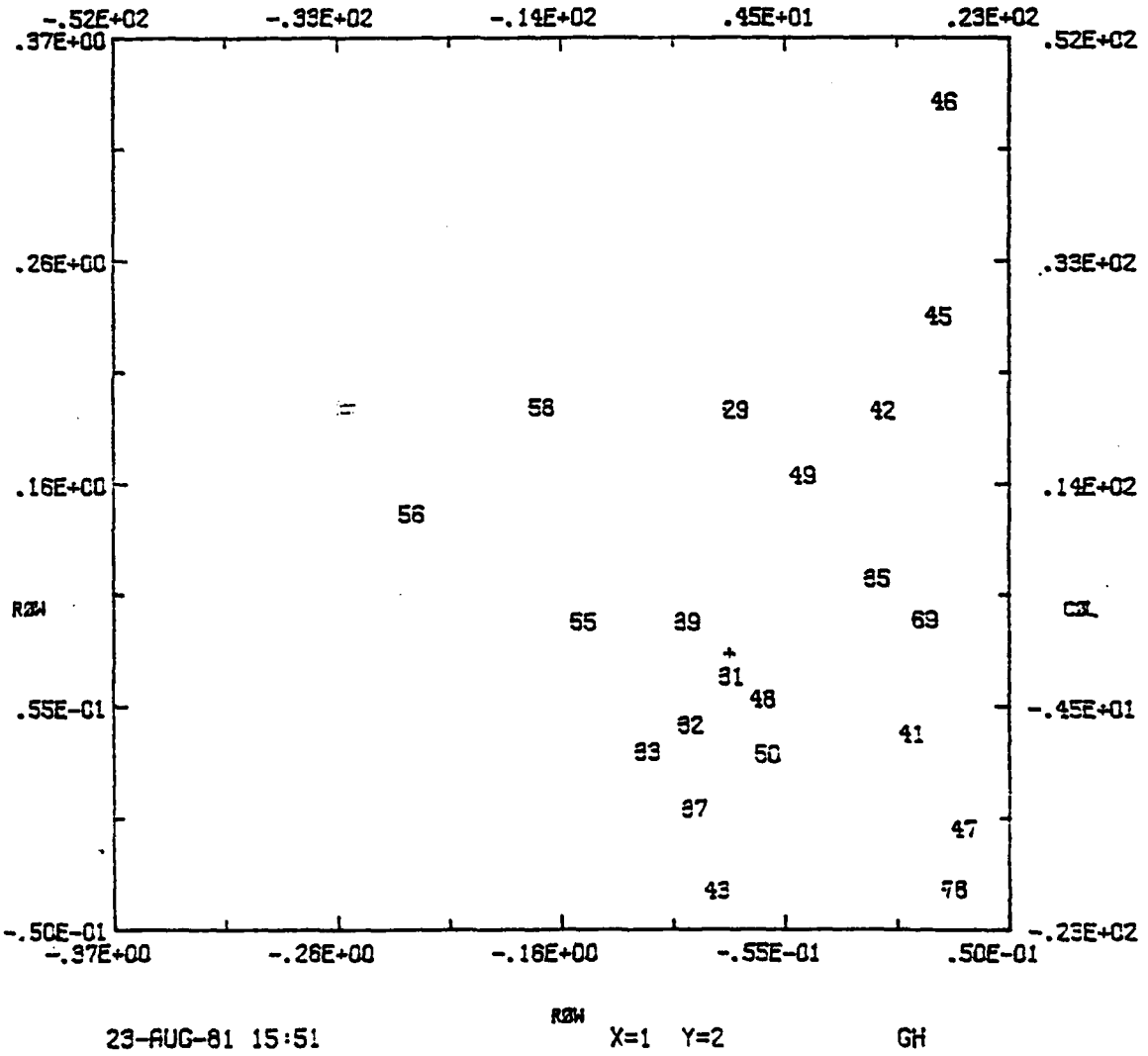



XL=-2.02E+00 XU= 2.02E+00 YL=-2.02E+00 YU= 2.02E+00 ZL=-2.02E+00 ZU= 2.02E+00
Y
Z
BLANK
X

Display 7.11a: RALL, WINDOW, MULTI. Restore display of all row markers, display a subset of the plot area using three orthogonal views.

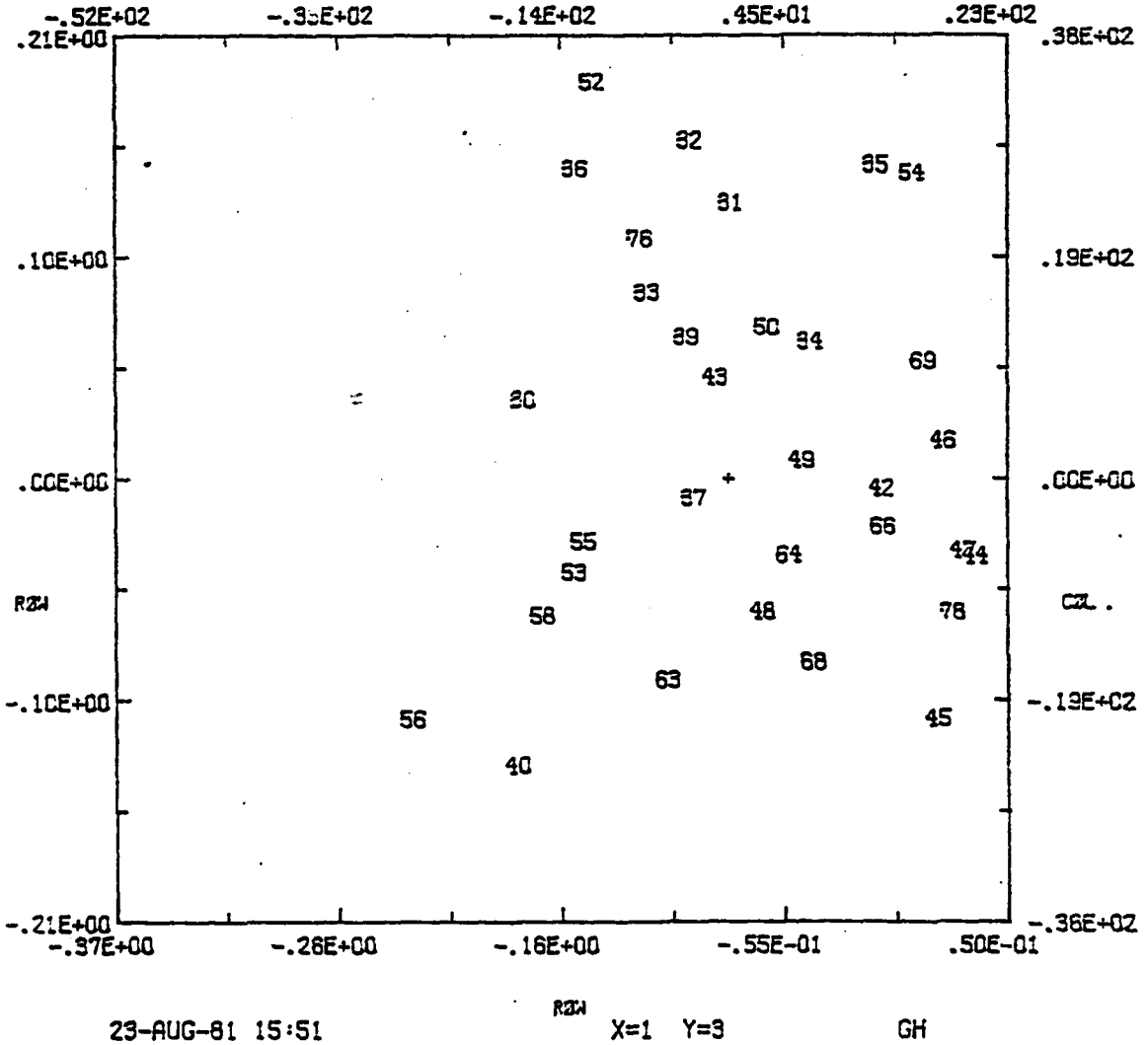
```
B:RALL
ALL ROW MARKERS WILL BE INCLUDED WHEN ROW MARKERS ARE EMPLOYED.
B:WINDOW
THE NEXT PLOT COMMAND WILL REQUEST WINDOW INFORMATION.
B:MULTI    (Plots 7.8.1-7.8.3)
ENTER NUMBER OF COMPONENTS TO BE USED:3
ENTER 3 COORDINATES(BETWEEN -.5 AND .5) TO CENTER THE WINDOW,
AND AN EDGELENGTH(BETWEEN 0 AND 1).
-.2 .2 0  .5
```

Display 7.11b: RALL, WINDOW, MULTI (View 1, coordinates 1 and 2).



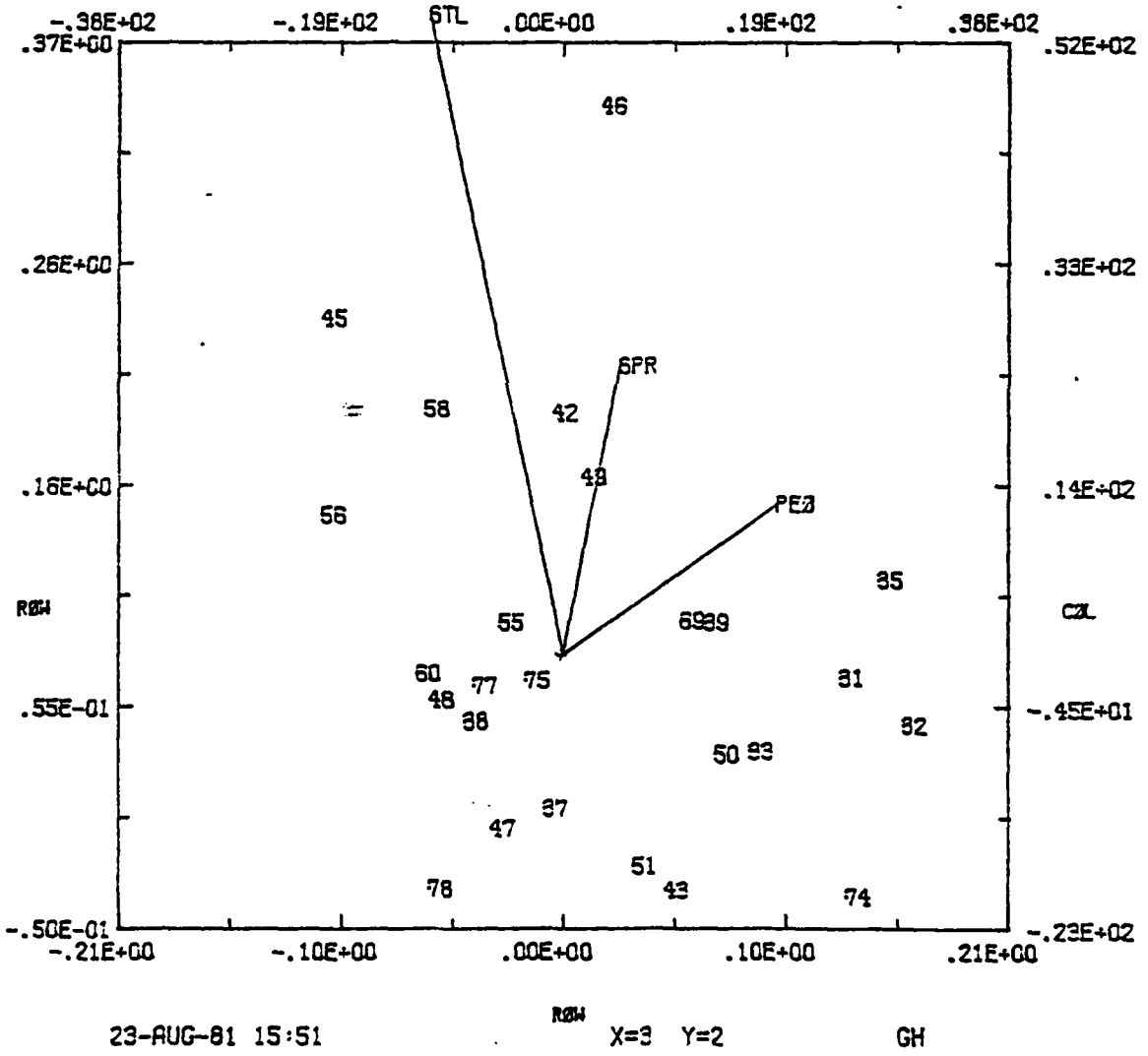
23-AUG-81 15:51 X=1 Y=2 GH

Display 7.11c: RALL, WINDOW, MULTI (View 2, coordinates 1 and 3).



23-AUG-81 15:51 X=1 Y=3 GH

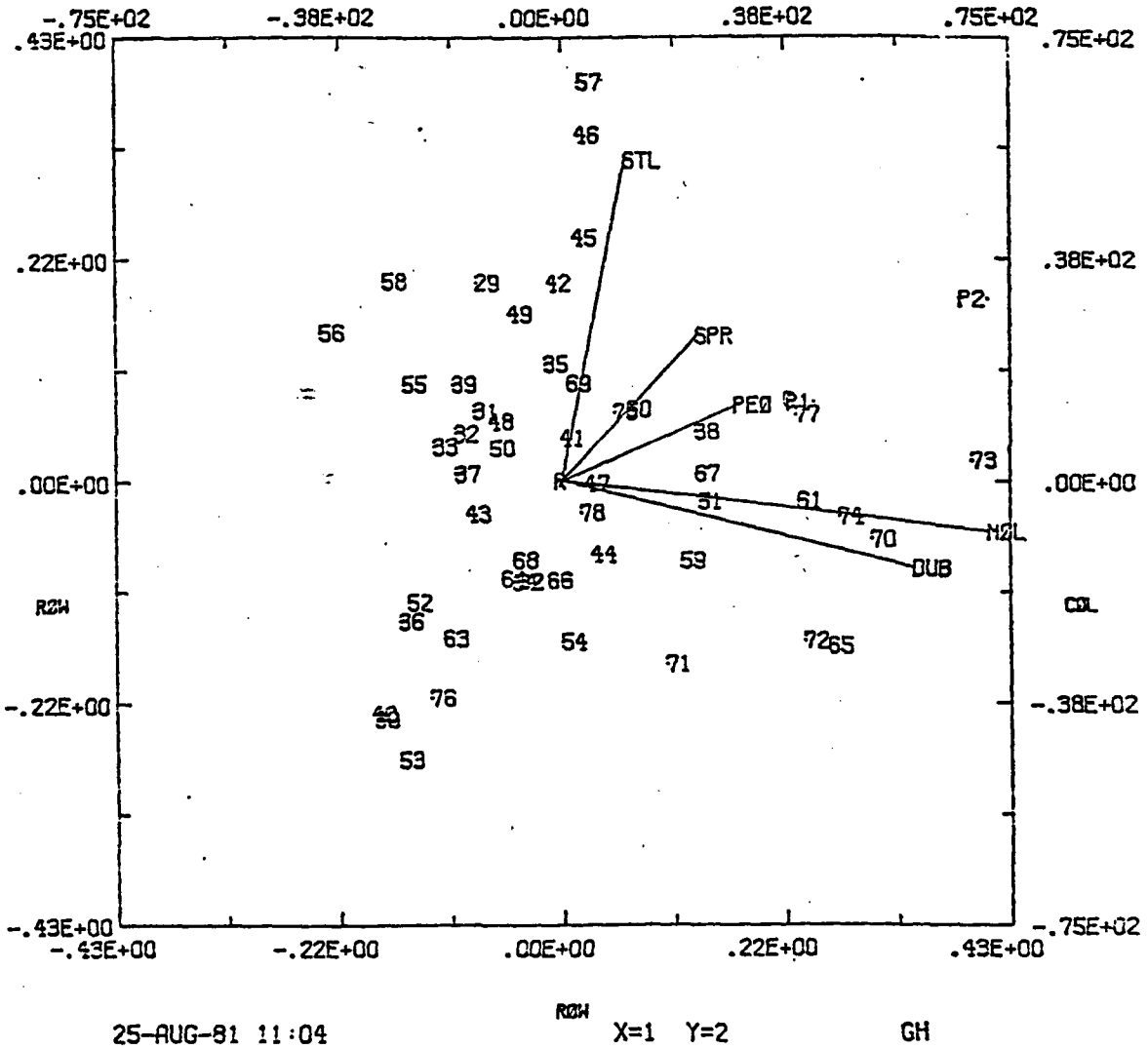
Display 7.11d: RALL, WINDOWN, MULTI (View 3, coordinates 2 and 3).



Display 7.12a: PROJECT, BIPLOT. Project two new data points on the biplot translating them before projection.

```
B:PROJECT
TYPE 1 TO PROJECT NEW DATA POINTS, OTHERWISE TYPE 0.
1
B:BIPLOT
ENTER THE NUMBER OF POINTS TO BE PROJECTED(MAX.=100).
2
ENTER 1 IF POINTS ARE TO BE INPUT FROM THE TTY,
OTHERWISE ENTER 0.
1
POINTS ARE ASSUMED  5 DIMENSIONAL. THEY WILL BE
READ POINT BY POINT. ENTER THEIR FORMAT.
(5F4.0)
DO YOU WISH TO TRANSLATE POINTS BEFORE PROJECTING(Y OR N):Y
ENTER THE  5 COORDINATES OF THEIR NEW ORIGIN.
38. 38. 35. 35. 38.
ENTER POINTS.
48. 59. 41. 40. 46.
57. 70. 47. 45. 54.
ENTER 2 COORDINATES(BETWEEN -.5 AND .5) TO CENTER THE WINDOW,
AND AN EDGELENGTH(BETWEEN 0 AND 1).
0 0 1
```

Display 7.12b: PROJECT, BIPLOT.

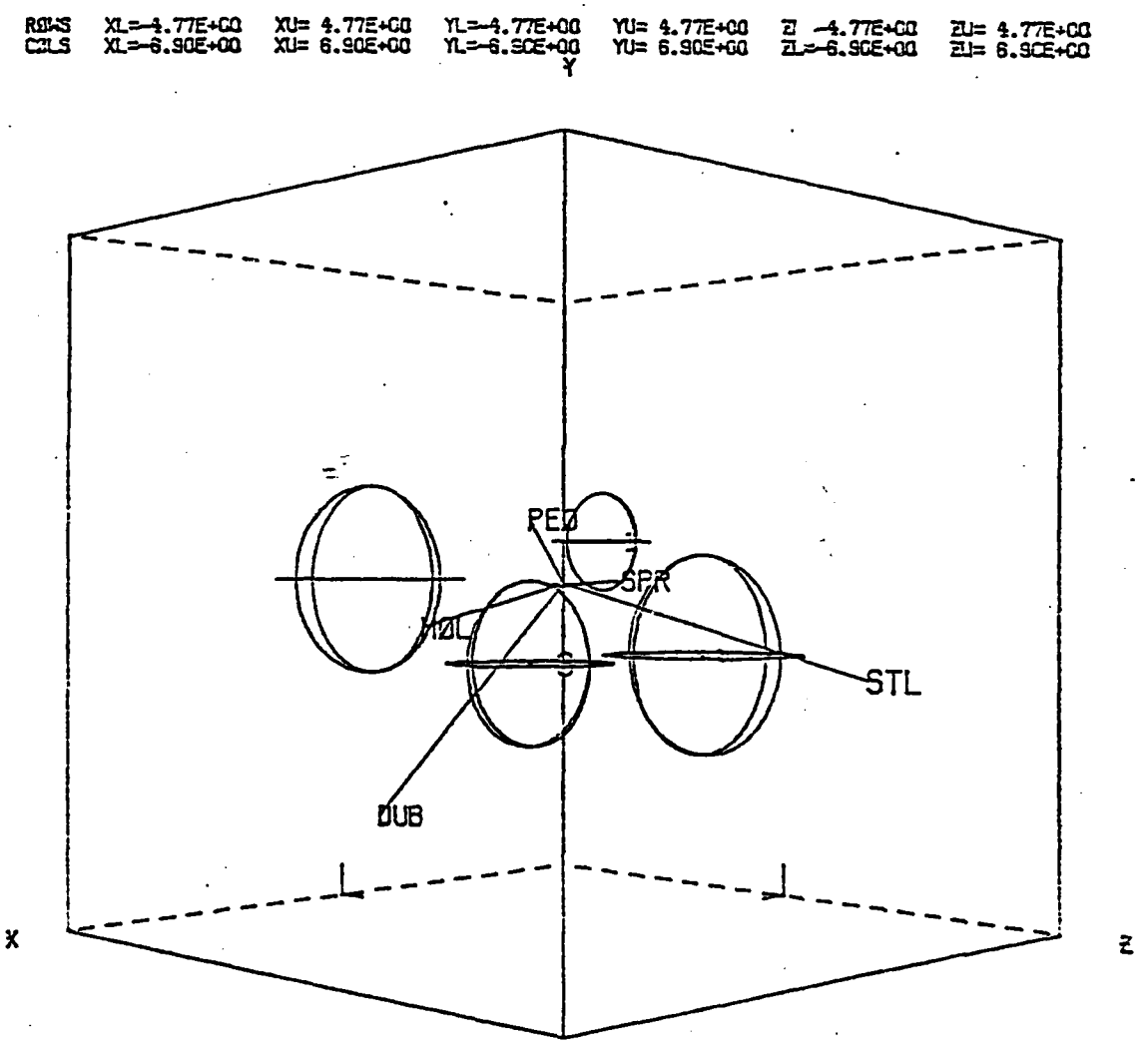


Note: New points are identified by labels P1 and P2.

Display 7.13a: READ, CVECTOR, CLABEL, CIRCLE, ROTATE, PERSP. Rotated display of MANOVA biplot with comparison spheres for each group.

```
B:READ
ENTER NAME OF PLQ FILE:
Unit=41  :/ACCESS=SEQINOU/MODE=ASCII

Enter new file specs. End with an $(ALT)
*MANOV.PLQ$

   3 COMPONENTS READ.


ENTER: 1 FOR GH FACTORIZATION,
       2 FOR JK FACTORIZATION,
       3 FOR GF FACTORIZATION,
       4 FOR EK FACTORIZATION,
       5 FOR SQRT FACTORIZATION.
2
B:CVECT
COLUMN MARKERS WILL BE PLOTTED AS VECTORS.
B:CLABE
CHOOSE LABELS FOR COLUMN MARKERS BY ENTERING:
1 FOR COLUMN NUMBER(PREFIXED BY C),
2 FOR RANKS ON SOME VARIABLE,
3 FOR DECILE RANKS MINUS 1(I.E. 0 TO 9),
4 FOR STANDARD SCORES ON SOME VARIABLE,
5 TO ENTER LABELS(MAX. LENGTH=5 CHARACTERS).
5
ENTER 1 IF LABELS OR LABELING VARIABLE ARE TO BE
ENTERED FROM THE TTY, OTHERWISE ENTER 0.
1
ENTER FORMAT FOR LABELS OR LABELING VARIABLE.
USE A-FORMAT)
(A3)
ENTER LABELING VARIABLE OR LABELS(    5 VALUES OR
LABELS IN ALL):
DUB
MOL
PEO
SPR
STL
B:CIRCLE
TYPE 1 TO DRAW CIRCLES OR SPHERES, OR 0 TO OMIT THEM.
1
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
1
ENTER 3 ANGLES(IN DEGREES)TO ROTATE THE CUBE COUNTER-
CLOCKWISE ABOUT AXES THROUGH THE CENTER OF THE CUBE,
FIRST ABOUT THE X-AXIS,THEN THE Y-AXIS AND FINALLY THE Z-AXIS.
0 -45 0
B:PERSP    (Plot 7.10)
TO CONTINUE AFTER A PLOT IS DISPLAYED, TYPE (CR).
TYPE R TO REUSE PREVIOUSLY ENTERED RADII,
     E TO ENTER RADII, OR
     A TO ABORT THIS OPTION.
E
ENTER   4 RADII FOR COMPARISON SPHERES.
.67784 1.41103 1.15210 1.15210
```

Display 7.13b: READ, CVECTOR, CLABEL, CIRCLE, ROTATE, PERSP.



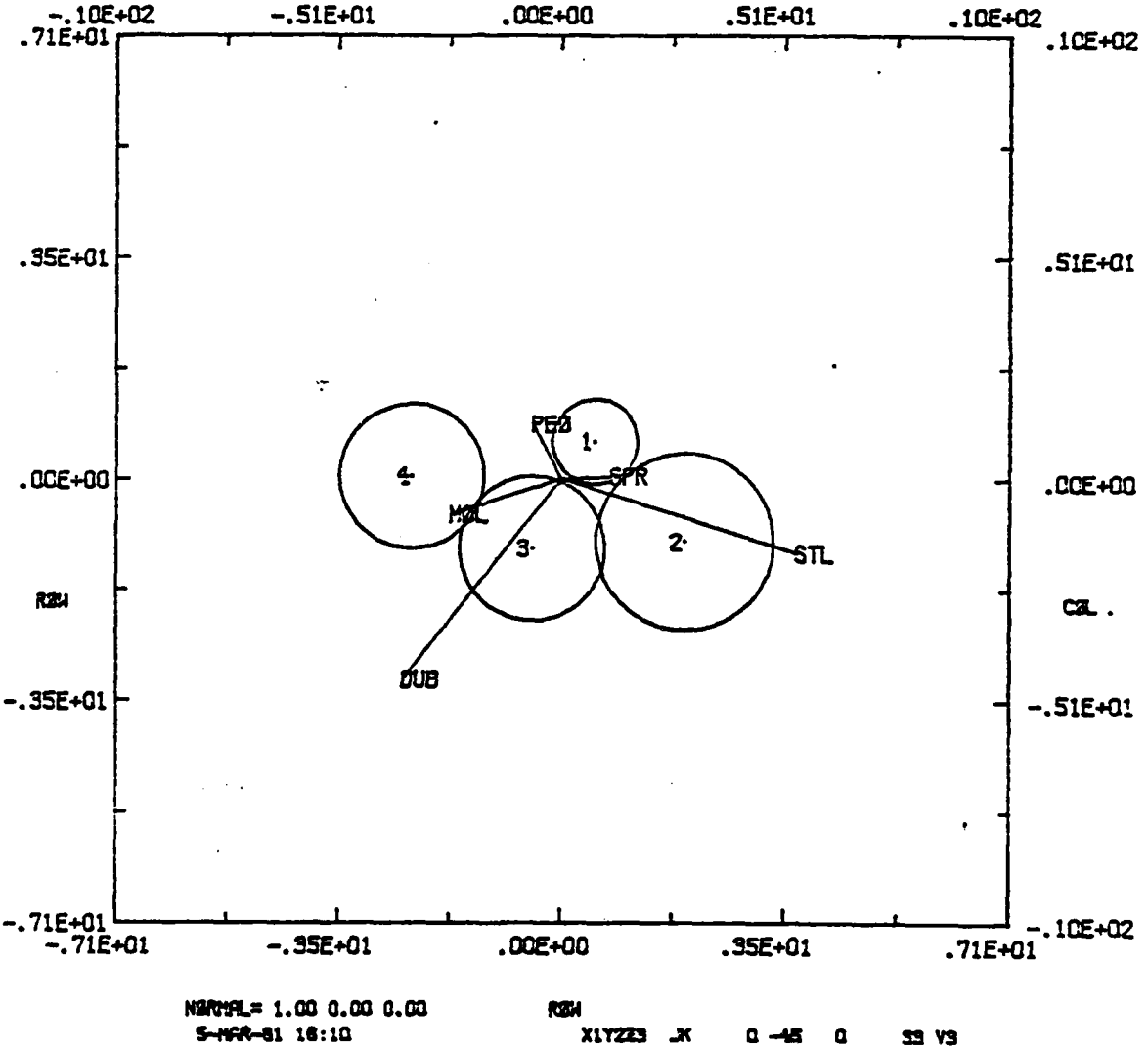
Note: MANOVA biplot is identified as a JK biplot.

AD-A114 449 ROCHESTER UNIV NY DIV OF BIOSTATISTICS F/G 9/2
BGRAPH -- A PROGRAM FOR BIPLOT MULTIVARIATE GRAPHICS. VERSION 1--ETC(U)
SEP 81 M C TSIANCO, C L ODOROFF, S PLUMB N00014-80-C-0387
UNCLASSIFIED TR-81/20 NL

Display 7.14a: ROTATE, SIDE. Display of a projection of the MANOVA bimodel on the plane of observation after a 45° rotation about the y-axis.

```
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
1
ENTER 3 ANGLES(IN DEGREES)TO ROTATE THE CUBE COUNTER-
CLOCKWISE ABOUT AXES THROUGH THE CENTER OF THE CUBE,
FIRST ABOUT THE X-AXIS,THEN THE Y-AXIS AND FINALLY THE Z-AXIS.
0 -45 0
B:SIDE   (Plot 7.11)
TYPE R TO REUSE PREVIOUSLY ENTERED RADII,
     E TO ENTER RADII, OR
     A TO ABORT THIS OPTION.
R
ENTER MAGNIFICATION FACTOR.
1
```

Display 7.14b: ROTATE, SIDE

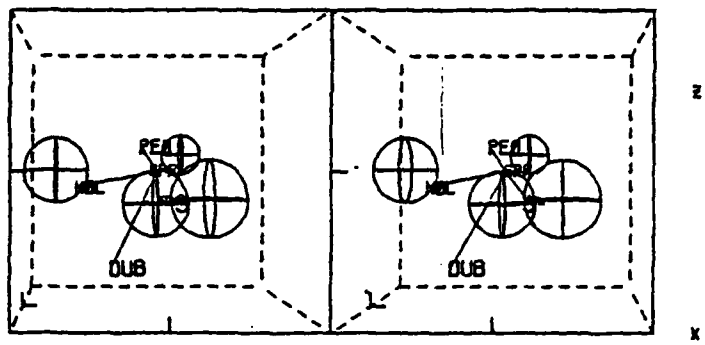


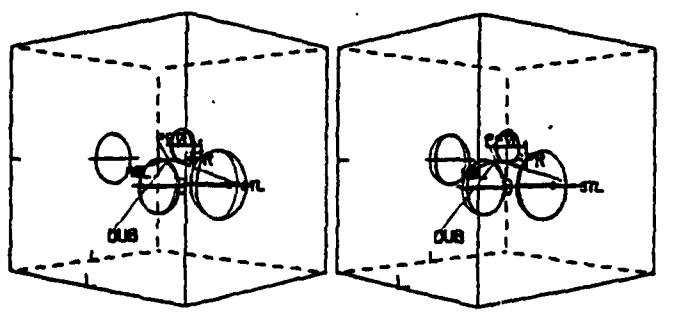
Note: The NORMAL is with respect to the new axes, after rotation.

Display 7.15a: ROTATE, STEREO. Rotated stereo display of MANOVA biplot. Observer at (0, 0, -3).

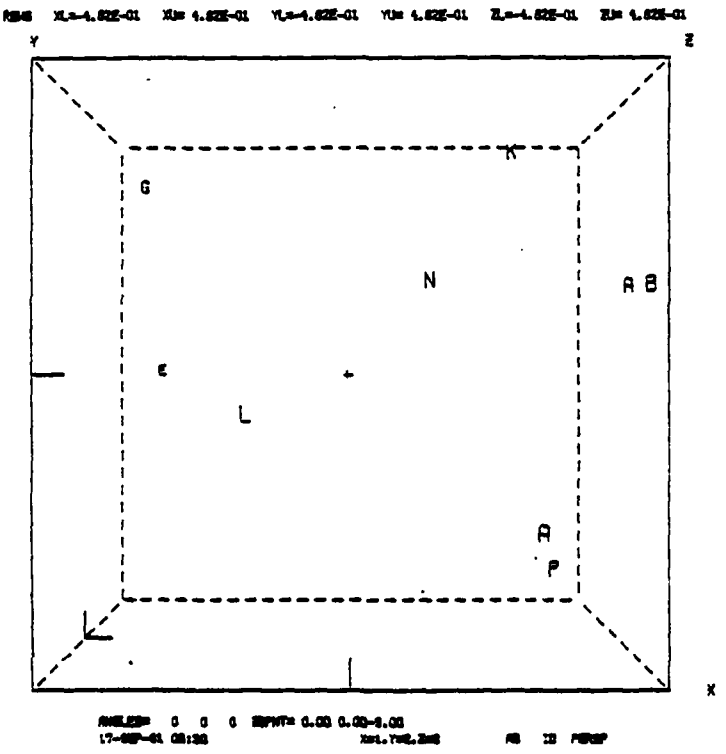
```
B:ROTATE
TYPE 1 TO SPECIFY ROTATION WITH 3 ANGLES, OR 2 TO
GIVE AN AXIS AND AN ANGLE.
1
ENTER 3 ANGLES(IN DEGREES)TO ROTATE THE CUBE COUNTER-
CLOCKWISE ABOUT AXES THROUGH THE CENTER OF THIS CUBE,
FIRST ABOUT THE X-AXIS,THEN THE Y-AXIS AND FINALLY THE Z-AXIS.
0 -45 0
B:STERE   (Plot 7.12)
TYPE 1 TO PLOT BOTH VIEWS ON 1 PAGE,
     2 TO PLOT THEM SEPARATELY, OR
     3 TO PAUSE BETWEEN VIEWS OF HARD COPY ANALGLYPHS.
1
TYPE R TO REUSE PREVIOUSLY ENTERED RADII,
     E TO ENTER RADII, OR
     A TO ABORT THIS OPTION.
R
```

Display 7.15b: ROTATE, STEREO.





Note: Stereo displays are a 65% followed by a 74% reduction on a Xerox 9200 copier. Note separation of PEO.


Y
Z
X
G
K
N
A B
E
L
A
P

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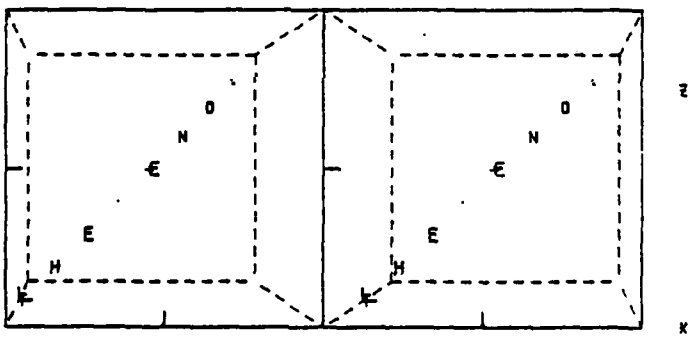
XL=-4.00E+00 XU= 4.00E+00 YL=-4.00E+00 YU= 4.00E+00 ZL=-4.00E+00 ZU= 4.00E+00
Y
Z
Y
Z
K